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Acceptance sampling plans using hyper-geometric theory for finite population under Q-Weibull distribution

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In this paper, truncated lifetime testing is considered, and new single acceptance sampling plans (SASP) are proposed assuming that the lifetime distribution is the q-Weibull distribution of a product. Assuming a finite population size (limited population size, N); the inspection process for a single sampling plan is ASP(n,p) begins by choosing a simple random sample from a given lot, then based on pre-assigned quality standards, the manager will decide to reject the lot if some items fail to meet the pre-assigned quality standards. Since the population is limited; then this experiment meets the hypergeometric distribution assumptions. The hypergeometric theory is applied to compute the probability of acceptance, and the procedure is used to compute the minimum sample size and the operating characteristics of the sampling plans. Also, a real data analysis is given to illustrate the applicability of the proposed plan in the industry.

keywords: Q-Weibull Distribution, Single Acceptance Sampling Plan, Reliability Testing, Operating Characteristic Function. (three to six keywords separated by comma).

1 Introduction

Acceptance sampling (AS) is one of the main fields in planning for quality assurance, which uses statistical methods in quality control. It is inspection and decision-making

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concerning products based on a randomly selected sample. Acceptance sampling plans (ASP) are sequential sampling procedures that are typically used as a tool for product inspection to test if the product is within the specification limits or not before consumer's use (Al-Nasser and Obeidat (2020); Al-Nasser and Haq (2021); Tripathi, Dey, and Saha (2021)). The US Military for testing shipment bullets was the first that used ASP during World War II. Nowadays, ASP is considered as significant statistical procedures that are used to assist companies, manufacturing, and educational institutions to minimize the product variability and to improve the outgoing product quality (Montgomery (2009); Aslam and Ali (2019); Gogah, and Al-Nasser (2018); Saha, Tripathi, and Dey (2021); Tripathi, Saha, and Alha (2020)). ASP help us to examine whether the manufactured products meet the pre-specified quality levels, and to assess the quality level of the product based on sampled items. Since the sample is randomly selected from a lot that includes good and bad items, there is always a causal of making a wrong decision. And there are two types of risks (Aydemir and Olgun , 2010): producer's risk (α^*) which is probability of rejecting a good lot and consumer's risk (β^*) which is the probability of accepting a bad lot. Henceforth, the operation of the single ASP has parameters:

- The sample size or the number of units on the test (n),
- Let μ and μ_0 be the true and the specified average product life, respectively
- An acceptance number c, number of acceptable failures or defective items (d).
- The maximum test duration time, t_0.
- Decide on the following hypothesis and declare a lot of product is good or bad:

$$H_0 : \mu \geq \mu_0 \text{ (The product is good)}$$
$$H_1 : \mu < \mu_0 \text{ (The product is bad)}$$

Then the decision on the hypothesis can be taken in one of two ways: reject the null hypothesis if D is more than c, or truncate the experiment as soon as the time of the experiment exceed to, whichever occurs first (Aslam and Ali (2019); **?**); Al-Omari, Al-Nasser (2019)). Usually, the researchers develop ASP if the product's lifetime follows a specific lifetime distribution such as, exponential, Gamma, Weibull, etc. (Balakrishnan, Leiva, and Lopez (2007); Rao, Kantam, Rosaiah and Reddy (2012); Al-Omari, Al-Nasser (2019); Al-Omari (2015); Al-Nasser, Al-Omari, Bani-Mustafa, and Jaber (2018); Schilling, and Neubauer (2009)). Assuming the population size is large, and then by considering the binomial theory in developing the ASP, (Kantam and Rosaiah (2005); Baklizi, El Masri, and Al-Nasser (005)), developed the truncated life test when the life distribution of the test items follows an inverse Rayleigh distribution. Al-Omari, Al-Nasser and Ciavolino (2019) showed a new acceptance sampling plans based on Rama distribution in the particular case when the mean lifetime test is truncated. Gogah, and Al-Nasser (2018) used the median ranked acceptance sampling plans for the exponential distribution. In addition, Baklizi (2003) proposed a single ASP for Pareto model, whose he found the minimum sample size and the characteristics function for the model. Moreover, ASP is suggested based on the truncated life test when the life distribution of the test items follows an exponentiated Fréchet distribution (Al-Nasser, Al-Omari (2013)). . Also, Al-Omari, Aslam , Al-Nasser (2018) developed a single-acceptance sampling plan for a new lifetime distribution called an Ishita distribution, under the assumption that the mean lifetime is a pre-assigned quality parameter. In this article, we consider of using q-Weibull distribution. Accordingly, we organized the article in several sections, the next section will introduce the q-Weibull distribution and discuss some of its properties. Section 3, illustrate the idea of single acceptance sampling for finite populations, section 4 hypergeometric theory under the ASP context. In section 5 a real data analysis is given and the article is end up with some remarks given in section 6.

2 q-Weibull distribution

The q-Weibull distribution (QWD) can describe as complex systems with long-range interactions and long-term memory (Xu, Droguett, Lins, and Moura (2017); Picoli, Mendes, and Malacarne (2003)). The main idea of QWD obtained by the connection between Tsallis statistics and pathway idea of Mathai (2005). QWD is derived by maximizing the Tsallis entropy (Mathai, Saxena, and Haubold (2010)):

$$Tsallis_{Entropy} = \int_{-\infty}^{\infty} \frac{f(x)^q dx - 1}{1-q}; q \neq 1 \tag{1}$$

where q is a real parameter.

Subject to the probability density function constraint *i.e.*, $\int_{-\infty}^{\infty} f(x)dx = 1$; and a fixed mean constraint $\int_{-\infty}^{\infty} xf(x)dx = given$; assuming non-negative domain. Then the corresponding QWD is obtained by parametrizing the generalized pathway model, where the parameter q is restricted to $q < 1$; or $1 < q < 2$. Accordingly, the QWD's probability density function (pdf) is:

$$f_q(x) = \alpha\lambda^\alpha(2-q)x^{\alpha-1}exp_q(-(\lambda x)^{-\alpha}); x \geq 0 \tag{2}$$

such that $x \in \begin{cases} [0,\infty) & , for\ 1 < q < 2, \\ [0, x_{\max}] & , for\ q < 1, \end{cases}$ with $x_{\max} = \frac{\alpha}{(1-q)^{\frac{1}{\beta}}}$

where $\alpha > 0$ *and* $q < 2$; both are shape parameters, and $\lambda > 0$ **is a scale parameter. Moreover, t**he q-type functions introduced in non-extensive statistical mechanics for non-extensive formalism. One of the functions is the q-exponential function (deformed exponential) defined as (Xu, Droguett, Lins, and Moura (2017))

$$\exp_q(x) = \begin{cases} (1+(1-q)x)^{\frac{1}{1-q}} & , if\ 1+(1-q)x > 0, \\ 0 & , otherwise \end{cases}$$

Henceforth final formula of the QWD will be:

$$f_q(x) = \alpha\lambda^{\alpha}(2-q)(x)^{\alpha-1}\left[1+(q-1)(\lambda x\lambda x\lambda x_{\lambda x)^{\alpha}}{}^{-\frac{1}{q-1}} \quad {\scriptstyle ,\ x>0,\ \alpha,\lambda>0\ ,1<q<2}\right.$$

The corresponding q-Weibull cumulative distribution function (CDF) as follows:

$$\mathrm{F}\,(x) = \int_0^x f\,(t)dt = 1 - [1+(q-1)(\lambda x)^{\alpha}]^{\frac{q-2}{q-1}} \;\; ; \;\; x \geq 0$$

As $q \to 1$ tended to be Weibull density with two parameters α and λ and the p.d.f. is

$$\mathrm{f}\,(t) = \alpha\lambda^{\alpha}\,(t)^{\alpha-1}\,e^{(\lambda x)^{\alpha}}\ ,\ x \geq 0,\ \alpha, \lambda > 0$$

when $\alpha = 1$, becomes q-exponential distribution

$f\,(x) = \lambda\,(2-q)\,[1+(q-1)\,\mathrm{x}]^{\frac{1}{1-q}}\ ,\ x \geq 0, \lambda > 0$
and when $q \to 1$ with $\alpha = 1$ became the standard exponential density. Moreover, the survival function

$\overline{\mathrm{F}}\,(x) = 1 - F\,(x)$ =1-$\left[1 - [1+(q-1)(\lambda x)^{\alpha}]^{\frac{q-2}{q-1}}\right]$
=$[1+(q-1)(\lambda x)^{\alpha}]^{\frac{q-2}{q-1}}$; $x \geq 0$
However, the Hazard rate function is :

$$h\,(x) = \frac{f\,(x)}{\overline{\mathrm{F}}\,(x)} = \frac{\alpha\lambda^{\alpha}\,(2-q)\,(x)^{\alpha-1}\,[1-(1-q)\,(\mathrm{x})^{\alpha}]^{\frac{1}{1-q}}}{[1-(1-q)\,(\mathrm{x})^{\alpha}]^{\frac{2-q}{1-q}}}$$

$$= \frac{\alpha\lambda^{\alpha}\,(2-q)\,(x)^{\alpha-1}}{1-(1-q)\,(\mathrm{x})^{\alpha}} \;\; ; |x| < \frac{1}{\lambda(1-q)^{\frac{1}{\alpha}}}$$

The hazard rate function is decreasing for both $q < 1$and $1 < q < 2$ and for $q = 1$, it is decreasing, increasing, and constant based on the value of $\alpha <\ 1$,$\alpha >\ 1$, or $\alpha =\ 1$; respectively.
Moreover, the moment generating function of QWD is

$$M_X\,(t) = E\left(e^{\mathrm{tX}}\right) = \int_0^{\infty} e^{\mathrm{tx}}\alpha\lambda^{\alpha}\,(2-q)\,(x)^{\alpha-1}\,[1-(1-q)\,(\mathrm{x})^{\alpha}]^{\frac{1}{1-q}}\mathrm{dx}$$

Then k^{th} moment about of QWD is:

$$E\left(x^k\right) = \frac{2-q}{\lambda^k\,(q-1)^{\frac{k}{\alpha}+1}}\frac{\Gamma\left(\frac{k}{\alpha}+1\right)\Gamma\left(\frac{1}{q-1}-\frac{k}{\alpha}-1\right)}{\Gamma\left(\frac{1}{q-1}\right)} \;\; ; \;\; \frac{1}{q-1} - \frac{k}{\alpha} - 1 > 0$$

Therefore, the mean and variance of the QWD are:

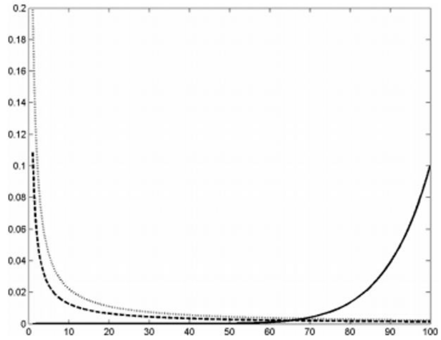

Figure 1: The hazard rate function.

$$\mu = E(x) = \frac{2-q}{\lambda (q-1)^{\frac{1}{\alpha}+1}} \frac{\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma\left(\frac{1}{q-1}-\frac{1}{\alpha}-1\right)}{\Gamma\left(\frac{1}{q-1}\right)} \; ; \; \frac{1}{q-1}-\frac{1}{\alpha}-1>0, \qquad (2.2)$$

$$\sigma^2 = E\left(x^2\right) - \left[E(x)\right]^2$$

$$= \frac{1}{{}^2(q-1)^{\frac{2}{\alpha}}\Gamma\left(\frac{2-q}{q-1}\right)} \left\{ 2\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(\frac{2-q}{q-1}-\frac{2}{\alpha}\right) - \frac{1}{\alpha}\frac{\Gamma^2\left(\frac{1}{\alpha}\right)\Gamma^2\left(\frac{2-q}{q-1}-\frac{1}{\alpha}\right)}{\Gamma\left(\frac{2-q}{q-1}\right)} \right\} ; \frac{2-q}{q-1}-\frac{1}{\alpha}>0 \; ,$$
(2.3)

3 Acceptance Sampling Plans for Finite Populations

The acceptance sampling experiment from finite population is performed by selecting a random sample of n items without replacement from a lot ofN items in which the lot contains D nonconforming or defective items. Now, if N is finite then D is assumed to be a random variable following the hypergeometric distribution with probability mass function:

$$P(D=x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} ; x = 0,1,\ldots,\; n \;,\; where\; M \;=[N*p]\,,\; 0<p<1 \quad (3.1)$$

where the lot quality $p = M/N$ and the sample size is n. Note that the number of good parts in the lot is$(N - M)$, and the number of good parts in the sample is $(n - x)$. Then the probability of acceptance is simply the probability that Dis less than or equal (acceptable number) is known by the operating characteristics (OC(p)) of the plan:

$$L(p) = P(D \le c)$$

Therefore, for a given values of c and β^* (Type II error), the objective is to determine the smallest sample size n such that:

$L(AQL) \leq \beta^*$. **(3.2)**

where AQL is the acceptable quality level that equivalent to the cumulative distribution for product characteristic under inspections (Baklizi (2003); Baklizi and El Masri (2004)). Moreover, based on the minimum sample size obtained by solving (3.1), researchers also will use the lot tolerance percent defective (LTPD) for finding the termination time by solving (3.2) for some given values of α^* (Type I error):

$$LTPD \geq 1 - \alpha^* \text{ . } (3.3)$$

4 ASP using QWD with finite population size

Single acceptance sampling plan ASP (n, p) will be designed assuming that the QWD is the lifetime distribution of a product; from finite population (limited population size, N). The inspection process for a single $ASP(n, p)$ begins by choosing a simple random sample from a given lot, based on pre-assigned quality standards, the manager will decide to reject the lot if some items fail to meet the quality standards pre-assigned. Since the population is limited; then this experiment meets the hypergeometric distribution assumptions. The acceptance sampling producer specifies the values of c and n which reduces level the probability of error to an acceptable level. In other words, the intention of acceptance sampling aims to minimize the probability of classifying the lot wrongly. When the sample contains a small number of defective products ($d \leq c$), this shows the high-quality lot to the consumer. Then the decision about the sampling plan will be If $d \leq c$ then accept the lot as likely high quality. However, if $d > c$ then reject the lot as likely low unacceptable quality. Moreover, the life test ends at a pre-assigned time t and the numbers of failures through this time interval $[0, t]$ are observed. For simplicity, we set the determined time t as

$$t = d\mu_0$$

where μ_0 is the specified mean lifetime and d is a positive constant (Baklizi (2003); Gui, and Aslam (2015); Al-Omari (2015)).

The acceptance sampling plan (n, c, d) consists of:

- The number of unit's n on test.
- The acceptance number c, where the lot is accepted if at most c failures out of n are observed at the end of the predetermined time t.
- A ratio $= \frac{t}{\mu_0}$.

The acceptance or rejection of the lot is equivalent to the acceptance or rejection of the hypothesis H_0:$\mu = \mu_0$.

Accordingly, the consumer's risk is fixed not to exceed $1-p^*$, $(0<p^*<1)$. That is the true mean lifetime μ is less than the specified mean lifetime μ_0 $(\mu<\mu_0)$. Therefore, the customer identifies the LTPD as the maximum acceptable rate of bad quality. If well thought out, the sampling plan has values for n and c which show little chance of acceptance if p is more than the LTPD the consumer tolerated. This means that the probability of error $P\,(x\le c|p\ge LTPD)=\beta$ is very small. In other words, the customer protects himself from against poor quality by selecting a suitable sampling plan that holds the risk for undesirable levels of p at a low and tolerable level. Therefore, the hypergeometric distribution could be applied and for given value c and p^*, the objective is to determine the smallest sample size n such that:

$$\sum_{d=0}^{c}\frac{\begin{pmatrix}M\\d\end{pmatrix}\begin{pmatrix}N-M\\n-d\end{pmatrix}}{\begin{pmatrix}N\\n\end{pmatrix}}\le 1-p^*,$$

where $p=F(t;\lambda_0)$ is the given cumulative distribution for product characteristic under inspections which depends only on the ratio $d=\frac{t}{\mu_0}$, and λ_0 is a value of λ when $\mu=\mu_0$.

4.1 Optimal sample size of the ASP $(n,c,\frac{t}{\mu_0})$

The optimal sample size for the ASP can be obtained under the following assumptions:

- *The confidence level of rejecting a lot if $\mu_0<\mu$ with probability p^* is, $1-p^*$ where $p^*\in(0,1)$.*
- *The lot size N is small and satisfies the hypergeometric distribution assumptions.*

Then, the problem will be in finding the optimal sampling plan ASP(N,n,c,$\frac{t}{\mu_0}$ is to find the minimum sample size n such that the number of failures d does not exceed c, to ensure that $\mu_0<\mu$ satisfies the following inequality:

$$\sum_{d=0}^{c}\left(\frac{\begin{pmatrix}M\\d\end{pmatrix}\begin{pmatrix}N-M\\n-d\end{pmatrix}}{\begin{pmatrix}N\\n\end{pmatrix}}\right)\le 1-p^*\ ,$$

$$\sum_{d=0}^{c}\frac{\begin{pmatrix}N*F\left(\frac{t}{\mu_0}*\frac{\mu_0}{\mu}\right)\\d\end{pmatrix}\begin{pmatrix}N-N*F\left(\frac{t}{\mu_0}*\frac{\mu_0}{\mu}\right)\\n-d\end{pmatrix}}{\begin{pmatrix}N\\n\end{pmatrix}}\le 1-p^*,$$

where $p=F\left(t,\mu_0\right)=1-\left[1+\left(\frac{t}{\mu_0}\frac{2-q}{(q-1)^{\frac{1}{\alpha}}}\frac{\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma\left(\frac{1}{q-1}-\frac{1}{\alpha}-1\right)}{\Gamma\left(\frac{1}{q-1}\right)}\right)^{\alpha}\right]^{\frac{q-2}{q-1}}.$

$$= 1 - \left[1 + \left(\frac{t}{\mu_0}\frac{2-q_0}{(q_0-1)^{\frac{1}{\alpha_0}}}\frac{\Gamma\left(\frac{1}{\alpha_0}+1\right)\Gamma\left(\frac{1}{q_0-1}-\frac{1}{\alpha_0}-1\right)}{\Gamma\left(\frac{1}{q_0-1}\right)}\right)^{\alpha_0}\right]^{\frac{q_0-2}{q_0-1}}, t \geq 0, \frac{1}{q-1} - \frac{1}{\alpha} - 1 > 0$$

The inequality becomes:

$$\sum_{d=0}^{c}\left(\frac{\binom{N * \left(1 - \left[1 + \left(\frac{t}{\mu_0}\frac{2-q_0}{(q_0-1)^{\frac{1}{\alpha_0}}}\frac{\Gamma\left(\frac{1}{\alpha_0}+1\right)\Gamma\left(\frac{1}{q_0-1}-\frac{1}{\alpha_0}-1\right)}{\Gamma\left(\frac{1}{q_0-1}\right)}\right)^{\alpha_0}\right]^{\frac{q_0-2}{q_0-1}}\right)}{d}\left(N - N * \left(1 - \left[1 + \left(\frac{t}{\mu_0}\frac{2-q_0}{(q_0-1)\cdots}\right.\right.\right.\right.}{\binom{N}{n}}\right.$$

$1 - P^*$

It could be noted that the given cumulative distribution of the product under inspection depend only on the ratio $d = \frac{t}{\mu_0}$, also the unknown parameters of the distribution which assuming to have the following initial values N=30, $\alpha_0 = 1, \lambda_0 = 2, q_0 = 1.2$ respectively when $\mu = \mu_0$ the experiment needs only to specify this ratio.

The minimum values of n satisfying inequality 3.3 were obtained for $P^* = 0.75,\ 0.9,\ 0.95,\ 0.99,$ and $\frac{t}{\mu_0} = 0.628, 0.942, 1.25,\ 1.571, 2.356, 3.141, 3.927, 4.712$. This choice is consistent with that of (Gupta, and Groll (1961); Kantam and Rosaiah (2005); Baklizi (2003); Tsai, and Wu (2006); Al-Nasser, Al-Omari (2013)). The results of the minimum sample size under q-Weibull distribution are given in Table.1. For example, assume that the researcher aims to ensure that the product's mean lifetime is at least 1000 h, with probability $p^* = 0.95$ when $c = 3$, such that the lifetime distribution test is at least t = 3141h; that is, t/μ_0 =3.141.Then from Table 1, the optimal sample size for this plan is 5. Accordingly, we can use the appropriated $ASP(n, c, t/\mu_0) = ASP\ (5, 3, 3.141)$, which means that a simple random sample of size 5 items should be selected from a lot of products, and if at most 3 items fail in meeting the quality standards before the specified time, t, with in the test period (1000 h), then the lot is accepted with probability 0.95.

4.2 Operating characteristic function of the ASP $(n, c, \frac{t}{\mu_0})$

Values of the $OC(p)$ as a function of $\frac{\mu}{\mu_0}$ for some selected sampling plans are given in Table 2 from the equation (3.2). The results summarize the values of the $OC(p)$ for the $ASP(30,\ n, c\ =\ 2, t/\mu_0)$. As an example, when p* = 0.99, the $OC(p)$ values for the $ASP(30,\ 9, 2, 0.942)$ are re-calculated as follows:

$\frac{\mu}{\mu_0}$	2	4	6	8	10	12
$OC(p)$	0.1298	0.6569	0.8568	0.9310	0.9793	1.0000

This means that if the true mean life is twice the specified mean life ($\frac{\mu}{\mu_0} = 2$) the producer's risk is about(1-0.1298 = 0.8702). Which means, using the QWD minimize the producer's risk, this is considering an advantage of QWD in acceptance sampling plan applications.

4.3 Producers risk of the ASP $(n, c, \frac{t}{\mu_0})$

The producer risk is the probability of rejecting the lot when $\mu \geq \mu_0$. Using the sampling plan under consideration, and given a value for the producer's risk, say 0.05, one may be interested in knowing what the value is of $\frac{\mu}{\mu_0}$ that will ensure the producer's risk less than or equal to 0.05. This value of $\frac{\mu}{\mu_0}$ is the smallest number $\frac{\mu}{\mu_0}$ for which $F\left(\frac{t}{\mu_0} * \frac{\mu_0}{\mu}\right)$ satisfying the inequality $L\,(\text{LTPD}) \geq 1 - \alpha^*$

$$\sum_{i=0}^{c} \frac{\binom{M}{d}\binom{N-M}{n-d}}{\binom{N}{n}} \geq 0.95.$$

Are computed and presented in Table 3 with $\alpha_0 = 1, \lambda_0 = 2\ , q_0 = 1.2$. As an example, suppose we are using the ASP$(30,\ 9, 2, 1.571)$ with the consumer's risk equals to 10% ($p^* =\ 0.90$), then from Table 3, the minimum value of $\frac{\mu}{\mu_0}$ is 11.23. It implies that, when c = 2, the lot with 9 items will be rejected with probability less than or equal to 0.05.

5 Real data application

The data considered in this illustration is lifetime data measured in months of 20 small electric carts that used by manufacturing company for internal transportation and delivery services in a large manufacturing facility (Zimmer, Keats and Wang (1998); Lio, Tsai, and Wu (2010)): (0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53.0)

Now, suppose that the manufacturing company assured that the mean life of the electric carts is 18 months, then what is the acceptable sampling plan with $^* = 0.90$, for this manufacturing company?. The goodness of fit results was acceptable ($-2MLL = 73.6867$, $AIC = 151.3772$, $BIC = 153.3687$, $CAIC = 152.0831$, $HQIC = 151.766$, $A\text{–}D = 0.0746$, $K\text{–}S = 0.0584$, $P - value = 0.9999$). The results indicate that an excellent fit with K–S distance value between the empirical and the theoretical QWD equal to 0.1887329 with $p - value = 0.2075$. Moreover, the MLE is used to estimate the QWD parameters assuming that $\alpha = 1$. The results showed that $\widehat{q} = 0.9789$(0.2794) and $\widehat{\lambda} = 0.0655$(0.0359), where the values between brackets are the standard deviation of the estimator.

Therefore,

$$\widehat{\mu} = \frac{2-\widehat{q}}{\widehat{\lambda}\,(\widehat{q}-1)^{\frac{1}{\widehat{\alpha}}+1}} \frac{\Gamma\left(\frac{1}{\widehat{\alpha}}+1\right)\Gamma\left(\frac{1}{\widehat{q}-1}-\frac{1}{\widehat{\alpha}}-1\right)}{\Gamma\left(\frac{1}{\widehat{q}-1}\right)}$$

$= \frac{2-0.9789}{0.0655(0.9789-1)^{\frac{1}{1}+1}} \frac{\Gamma\left(\frac{1}{1}+1\right)\Gamma\left(\frac{1}{0.9789-1}-\frac{1}{1}-1\right)}{\Gamma\left(\frac{1}{0.9789-1}\right)} = 14.649$.

Also, it is assumed that $t = 18$ months. Therefore, $\frac{t}{\mu_0} = \frac{18}{14.649} = 1.257$ based on the estimated values and the given minimum sample size values in Table 1 with $p^* = 0.95$ and $\frac{t}{\mu_0} = 1.257$, is $n = 15$ when $c = 8$, therefore the optimal ASP will be ASP(20,15,8,1.257).

6 Conclusions

This study presents a new plan based on q-Weibull distribution. Assuming the products population is finite, the classical plan parameters are derived, including the minimum sample size, operating characteristic function producer risk and the minimum value of the true mean life. An illustration of the proposed plan is discussed using real data from the industry. Future work could be done by introducing sampling plans when the products population is infinite.

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Table 1: Minimum sample sizes necessary to ensure the mean life exceeds a given value μ_0 with probability P^* and the corresponding acceptance number c based on Hypergeometric probabilities when N =30 and q=1.2.

				$t/(\mu_0)$					
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	2	2	2	1	1	1	1	1
	1	5	4	3	3	2	2	2	2
	2	7	5	5	4	4	3	3	3
	3	9	7	6	6	5	5	5	4
	4	11	9	8	7	6	6	6	5
	5	13	11	9	8	7	7	7	6
	6	15	12	10	9	8	8	8	7
	7	17	14	12	11	9	9	9	9
	8	19	15	13	12	11	10	10	10
	9	21	17	15	13	12	11	11	11
	10	23	18	16	15	13	12	12	12
0.9	0	4	3	2	2	2	1	1	1
	1	6	5	4	3	3	3	3	2
	2	9	7	6	5	4	4	4	4
	3	11	8	7	6	5	5	5	5
	4	13	10	9	8	6	6	6	6
	5	15	12	10	9	8	7	7	7
	6	17	13	11	10	9	8	8	8
	7	19	15	13	12	10	9	9	9
	8	21	17	14	13	11	11	11	10
	9	23	18	16	14	12	12	12	11
	10	24	20	17	15	13	13	13	12
0.95	0	4	3	3	2	2	2	2	1
	1	7	5	4	4	3	3	3	3
	2	10	7	6	5	4	4	4	4
	3	12	9	8	7	6	5	5	5
	4	14	11	9	8	7	6	6	6
	5	16	13	11	9	8	7	7	7
	6	18	14	12	11	9	9	9	8
	7	20	16	13	12	10	10	10	9
	8	22	17	15	13	11	11	11	10
	9	23	19	16	15	13	12	12	11
	10	25	20	17	16	14	13	13	12
0.99	0	6	5	4	3	2	2	2	2
	1	9	7	6	5	4	3	3	3
	2	12	9	7	6	5	5	5	4
	3	14	11	9	8	6	6	6	5
	4	16	12	10	9	7	7	7	6
	5	18	14	12	10	9	8	8	7
	6	20	16	13	12	10	9	9	8
	7	21	17	14	13	11	10	10	9
	8	23	19	16	14	12	11	11	10
	9	24	20	17	15	13	12	12	11
	10	26	21	18	17	14	13	13	12

Table 2: OC function values for the Hypergeometric sampling plan (N=30,n,c,d) with given probability P^* for corresponding acceptance number c=2 based on QWD.

			$\frac{\mu}{\mu_0}$					
P^*	n	d	2	4	6	8	10	12
0.75	7	0.628	0.6569	0.9321	0.9914	1	1	1
	5	0.942	0.6327	0.9321	0.9781	0.9907	0.9975	1
	5	1.257	0.5	0.8568	0.9321	0.9781	0.9907	0.9975
	4	1.571	0.531	0.8736	0.9525	0.9819	0.9907	0.9962
	4	2.356	0.2835	0.7011	0.8736	0.931	0.9693	0.9819
	3	3.141	0.5015	0.799	0.9103	0.9594	0.9793	0.9862
	3	3.927	0.4335	0.7192	0.8621	0.9296	0.9594	0.9704
	3	4.712	0.2796	0.6207	0.799	0.8879	0.9296	0.9594
0.9	9	0.628	0.4411	0.8568	0.9793	1	1	1
	7	0.942	0.3257	0.8146	0.9321	0.9693	0.9914	1
	6	1.257	0.3257	0.7627	0.8799	0.9587	0.9819	0.9951
	5	1.571	0.3043	0.7548	0.898	0.9587	0.9781	0.9907
	4	2.356	0.2835	0.7011	0.8736	0.931	0.9693	0.9819
	4	3.141	0.1691	0.531	0.751	0.8736	0.931	0.9525
	4	3.927	0.1188	0.4072	0.6475	0.7965	0.8736	0.9048
	4	4.712	0.0394	0.2835	0.531	0.7011	0.7965	0.8736
0.95	10	0.628	0.3436	0.8088	0.9704	1	1	1
	7	0.942	0.3257	0.8146	0.9321	0.9693	0.9914	1
	6	1.257	0.3257	0.7627	0.8799	0.9587	0.9819	0.9951
	5	1.571	0.3043	0.7548	0.898	0.9587	0.9781	0.9907
	4	2.356	0.2835	0.7011	0.8736	0.931	0.9693	0.9819
	4	3.141	0.1691	0.531	0.751	0.8736	0.931	0.9525
	4	3.927	0.1188	0.4072	0.6475	0.7965	0.8736	0.9048
	4	4.712	0.0394	0.2835	0.531	0.7011	0.7965	0.8736
0.99	12	0.628	0.1869	0.6957	0.9458	1	1	1
	9	0.942	0.1298	0.6569	0.8568	0.931	0.9793	1
	7	1.257	0.1949	0.6569	0.8146	0.9321	0.9693	0.9914
	6	1.571	0.1531	0.6201	0.8254	0.9246	0.9587	0.9819
	5	2.356	0.102	0.5	0.7548	0.8568	0.9321	0.9587
	5	3.141	0.0413	0.3043	0.567	0.7548	0.8568	0.898
	5	3.927	0.0219	0.1912	0.433	0.6327	0.7548	0.8088
	4	4.712	0.0394	0.2835	0.531	0.7011	0.7965	0.8736

Table 3: Minimum value of the true mean life to specified mean life for the acceptance of a lot with producer's risk of 0.05 with q=1.2 using Hypergeometric Probabilities.

P^*	c	$\frac{t}{\mu_0}$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	24.595	36.892	49.229	61.526	92.269	123.012	153.794	184.537
	1	7.844	11.765	15.699	19.621	29.425	39.229	49.045	58.849
	2	3.649	5.474	7.304	9.128	13.689	18.25	22.817	27.378
	3	2.245	3.367	4.493	5.615	8.421	11.227	14.036	16.841
	4	1.731	2.596	3.464	4.33	6.493	8.656	10.822	12.985
	5	1.231	1.846	2.463	3.078	4.615	6.153	7.693	9.23
	6	1	1.351	1.802	2.253	3.378	4.503	5.63	6.755
	7	1	1	1.326	1.657	2.485	3.313	4.142	4.97
	8	1	1	1	1.058	1.586	2.114	2.643	3.171
	9	1	1	1	1	1.01	1.346	1.682	2.019
	10	1	1	1	1	1	1	1	1
0.9	0	24.595	36.892	49.229	61.526	92.269	123.012	153.794	184.537
	1	12.033	18.049	24.084	30.1	45.14	60.18	75.24	90.28
	2	4.489	6.734	8.985	11.23	16.84	22.451	28.069	33.68
	3	3.048	4.572	6.101	7.625	11.435	15.245	19.059	22.869
	4	2.245	3.367	4.493	5.615	8.421	11.227	14.036	16.841
	5	1.537	2.306	3.076	3.845	5.766	7.687	9.61	11.531
	6	1.107	1.66	2.215	2.768	4.151	5.534	6.919	8.302
	7	1	1.22	1.628	2.035	3.051	4.068	5.085	6.102
	8	1	1	1.071	1.338	2.006	2.674	3.343	4.012
	9	1	1	1	1	1.39	1.853	2.317	2.78
	10	1	1	1	1	1	1	1	1
0.95	0	24.595	36.892	49.229	61.526	92.269	123.012	153.794	184.537
	1	12.033	18.049	24.084	30.1	45.14	60.18	75.24	90.28
	2	5.748	8.621	11.504	14.378	21.562	28.746	35.939	43.124
	3	3.649	5.474	7.304	9.128	13.689	18.25	22.817	27.378
	4	2.597	3.895	5.197	6.496	9.741	12.987	16.236	19.482
	5	1.731	2.596	3.464	4.33	6.493	8.656	10.822	12.985
	6	1.231	1.846	2.463	3.078	4.615	6.153	7.693	9.23
	7	1	1.351	1.802	2.253	3.378	4.503	5.63	6.755
	8	1	1	1.326	1.657	2.485	3.313	4.142	4.97
	9	1	1	1	1.058	1.586	2.114	2.643	3.171
	10	1	1	1	1	1	1	1	1
0.99	0	24.595	36.892	49.229	61.526	92.269	123.012	153.794	184.537
	1	12.033	18.049	24.084	30.1	45.14	60.18	75.24	90.28
	2	7.844	11.765	15.699	19.621	29.425	39.229	49.045	58.849
	3	4.489	6.734	8.985	11.23	16.84	22.451	28.069	33.68
	4	3.048	4.572	6.101	7.625	11.435	15.245	19.059	22.869
	5	2.245	3.367	4.493	5.615	8.421	11.227	14.036	16.841
	6	1.731	2.596	3.464	4.33	6.493	8.656	10.822	12.985
	7	1.231	1.846	2.463	3.078	4.615	6.153	7.693	9.23
	8	1	1.351	1.802	2.253	3.378	4.503	5.63	6.755
	9	1	1	1.194	1.492	2.237	2.982	3.728	4.473
	10	1	1	1	1	1	1	1	1