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Bayesian statistical analysis of daily returns runs in Brazilian stock exchange

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In the analysis of financial market data, random variations in price movements can share non-trivial statistical properties as observed for the distributions of returns, with possibility of absence of autocorrelations in asset returns, volatility in blocks and presence of asymmetry between rises and falls of stock values. Asset price fluctuations are usually investigated using time series models to obtain inferences about interest rates and future forecasts. In this study we consider the use of existing probability distributions to model run lengths and absolute historical price run returns as an alternative to the use of usual time series models with applications for time series obtained from the NYSE stock exchange for three private banks located in the Brazil in the period from July 19, 2013, to July 19, 2018. We assume discrete Weibull distributions as an alternative to the exponential law commonly used in this type of analysis. Under this modeling approach it is possible to obtain information about the structure of the market such as the probability of the stock market rising and falling daily and the magnitude of the returns.

keywords: Bayesian analysis, daily runs, discrete models, financial market, private banks, stock exchange, Weibull distributions

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1 Introduction

In the financial market analysis, fluctuations in asset prices produce a financial time series where these series have been deeply investigated to get inferences and forecasting. In fact, if it is examined these series from a statistical point of view, according to Şensoy (2012), the random variations in price movements share non-trivial statistical properties. Most financial data researchers call those properties as stylized empirical facts where some are related to heavy tails in asset return distributions, absence of autocorrelations in asset returns, volatility clustering and asymmetry between rises and falls. In addition, as Ohira et al. (2002) and Li and Gao (2006) point out, the studies on the returns of stock exchange focus mainly on the analysis performed in a fixed established period of time such as hour, day or week. Moreover, another indicator for the movement of the financial market stock price is the sign of the return, that is, the rise or fall of stock prices.

In the last decades, empirical studies and the probabilistic structure of financial market stock price movements have attracted the interest of many researchers as observed in the literature. In this way, Safi and White (2017) used artificial neural networks for stock prices in Palestine considering short and long-term forecasting. Al Barghouthi et al. (2016) used unit root and Box-Jenkins models in the statistical analysis of the Amman Stock Exchange. Ramzan et al. (2012) considered the modeling and forecasting exchange rate dynamics in Pakistan using a ARCH (autoregressive conditional heteroscedasticity) family of models. Lau et al. (2018) studied exchange rate volatility in Cambodia, Laos, Myanmar and Vietnam (CLMV) using different approaches. Irfan et al. (2010) investigated the weak form efficiency of an emerging market by using parametric tests considering data from the Karachi stock market of Pakistan. Ismail and Awajan (2017) introduced a new hybrid approach EMD-EXP for short-term forecasting of daily stock market time series data. Chung and Zhang (2017) studied the volatility estimation using support vector machine with applications to major foreign exchange rates. De la Torre et al. (2018) used Markov-Switching models in Italian, British, U.S. and Mexican equity portfolios and presented a performance test. See also, Şensoy (2012), Adrian and Rosenberg (2008), Stockbridge (2008), Li and Gao (2006), Andersen et al. (2001), Cont (2001), Longin (1996), Tucker and Pond (1988), Akgiray and Booth (1987), among many others. For Akgiray and Booth (1987), empirical evidence indicates that the empirical distribution of stock returns shows severe deviations from normality which implies for the use of different long tail probability distributions to get good fit to the data which are implicit in reasonable economic scenarios. Two competing hypotheses stand out: the first, as Mandelbrot (1997) and Fama (1965) state that stock prices follow stable laws with characteristic exponents between one and two; and the second sees the empirical distributions of stock returns as long tail distributions with finite variances.

According to Şensoy (2012), the term used to measure the movement of prices referring to the return of market shares is "run". A run is a consecutive series of price movements without a sign reversal, hence a positive (negative) run is an uninterrupted sequence of positive (negative) returns and this run continues until a negative (positive) return comes out. As a special case, let us consider the daily closing historical prices of the

Oracle company in the period from February 2, 2015 to February 10, 2015 (excluding Saturdays and Sundays) given by 42.68, 43.04, 42.42, 43.16, 42.98, 43.40 and 43.99 with daily returns given by 0.019, 0.008, -0.014, 0.017, -0.004, 0.010, 0.014. The signs of those returns generate the sequence "+ + - + - + +" which contains three positive runs and two negative runs with lengths given by 2, 1 and 2 for positive runs and 1 and 1 for negative runs. The runs construction is relatively simple, but to the best of our knowledge, little research has been done on them in finance. In addition, for the analysis thereof, it is assumed that the lengths of the sequences of days on which the runs were positive (negative) for a certain company are independent and identically distributed. This assumption is necessary to estimate the parameters of chosen distribution using, for example, a Bayesian approach. Fama (1965) investigated the runs of several stocks, and found little evidence of efficiency violations based on series dependence on returns. Grafton (1981) conducted a similar survey to test the market efficiency hypothesis. Easley et al. (1997) used runs to examine dependence on intraday data.

The main goal of this paper is to explore the performance of discrete Weibull models (Nakagawa and Osaki, 1975) and continuous Weibull models, in the estimation of the distribution of run lengths and the distribution of the absolute returns under a Bayesian approach for three private banks located in Brazil from July, 19 2013 to July, 19 2018. The NYSE stock exchange was used to get the dataset. The Weibull distribution (Weibull, 1951) is one of the most popular distributions used to analyze positive observations, in particular considering lifetime data. Among the great advantages of the Weibull distribution, we can highlight its versatility and facility of use. The distribution provides a good fit for a wide range / variety of data sets (Lawless, 1982; Nelson, 2005). Discrete or continuous Weibull or generalized forms of the univariate or multivariate Weibull distribution have been extensively used in data analysis in many areas of interest as medicine, economy or engineering studies (see for example, Oliveira and Achcar, 2018; Freitas et al., 2018; Mdlongwa et al., 2017). Under a Bayesian approach assuming the Weibull distribution for the data, the posterior summaries of interest are obtained using standard MCMC (Markov Chain Monte Carlo) methods as the popular Gibbs Sampling algorithm (Gelfand and Smith, 1990) or the Metropolis-Hastings algorithm (Chib and Greenberg, 1995). The paper is organized as follows: in Section 2, it is presented some characteristics of the daily runs and the dataset. The both assumed Weibull distributions as well the estimation procedures using the Bayesian approach are also introduced in Section 2. In Section 3, it is presented the analysis and discussion of the distribution of run lengths and the distribution of absolute returns for the three private banks considered from July, 19 2013 to July, 19 2018, to illustrate the proposed methodology. Finally, Section 4 closes the paper with some concluding remarks.

2 Material and Methods

2.1 Runs of Daily Returns

For our analysis, it is considered daily closing values of three private banks from Brazil: Bradesco S.A. bank (BBD), Santander bank (BSBR) and Itau bank (ITUB). Our interest is to describe the distributions of run lengths and absolute run returns using a parametric model. The dataset was obtained from the historical price stated in NYSE stock exchange from July, 19 2013 to July, 19 2018.

2.2 Distribution of the Run Length and Absolute Returns

Before proceeding with the analysis, one concept should be well defined: *daily return*. According to Şensoy (2012), a daily return of an index is defined by,

$$r_t = \frac{s_t - s_{t-1}}{s_{t-1}} \tag{1}$$

where s_t is the index' closing value of day t. From those definitions, it could be obtained several informations from the empirical data. In this way, in Tables 1 and 2, it is presented the longest positive and negative runs with their corresponding date periods and their returns as well the total of positive and negative runs; and the frequencies of all runs with different lengths for each bank, respectively.

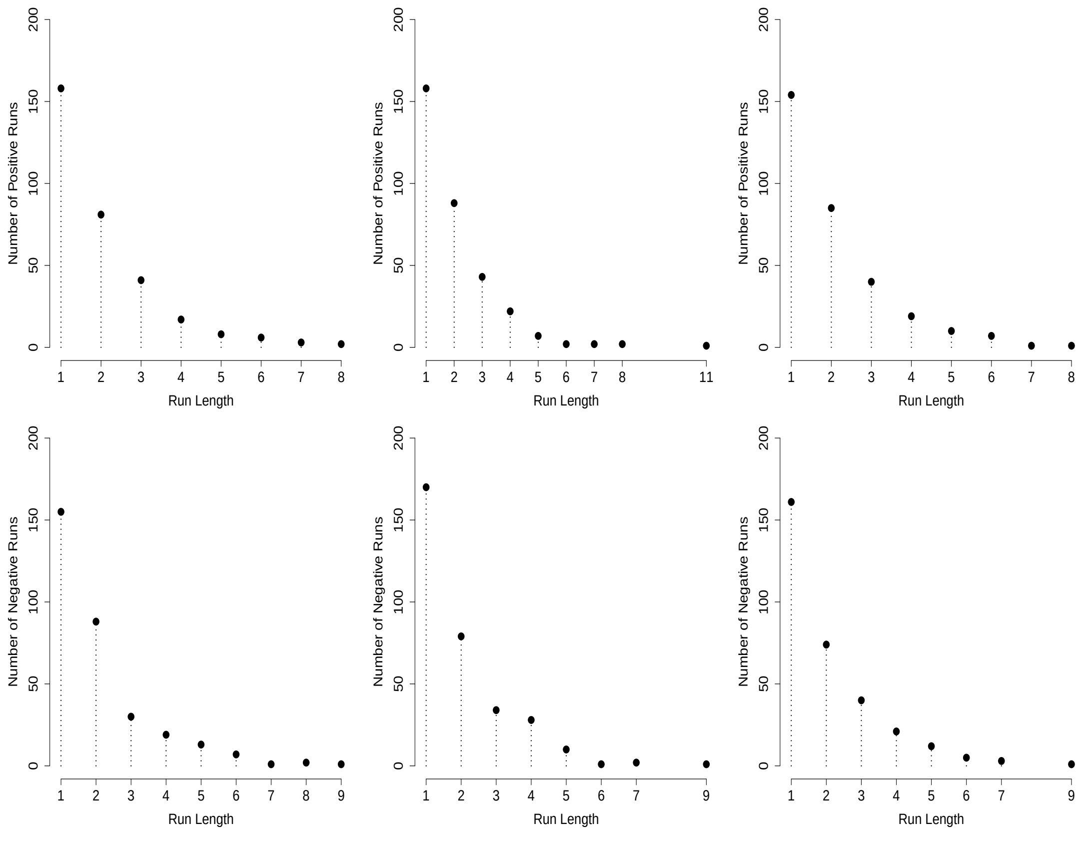

Figure 1: Run length distribution of historical price stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 (BBD → BSBR → ITUB).

Table 1: The longest positive (negative) runs of BBD, BSBR and ITUB in the period considered.

Bank	Longest Positive Run (mm/dd/yyyy)	Run Return	Total of Positive Runs
BBD	12/29/2015 - 01/11/2016 8 days	0.1695	316
	07/24/2015 - 07/14/2015 8 days	0.1621	
BSBR	12/28/2015 - 01/13/2016 11 days	0.2055	325
ITUB	04/27/2018 - 05/09/2018 8 days	0.1186	317
Bank	**Longest Negative Run (mm/dd/yyyy)**	**Run Return**	**Total of Negative Runs**
BBD	08/28/2013 - 09/11/2013 9 days	-0.1423	316
BSBR	09/28/2015 - 10/09/2015 9 days	-0.2809	325
ITUB	09/28/2015 - 10/09/2015 9 days	-0.2031	317

Table 2: Frequencies of runs with different lengths of BBD, BSBR and ITUB in the period considered.

Bank	Length (Positive Runs)											
	1	2	3	4	5	6	7	8	9	10	11	Total
BBD	158	81	41	17	8	6	3	2	-	-	-	316
BSBR	158	88	43	22	7	2	2	2	-	-	1	325
ITUB	154	85	40	19	10	7	1	1	-	-	-	317
Bank	**Length (Negative Runs)**											
	1	2	3	4	5	6	7	8	9	10	11	Total
BBD	155	88	30	19	13	7	1	2	1	-	-	316
BSBR	170	79	34	28	10	1	2	-	1	-	-	325
ITUB	161	74	40	21	12	5	3	-	1	-	-	317

From Tables 1 and 2, it could be seen that the negative runs has the longest run equals to nine and there is only one run for all banks. Most of negative runs are concentrated at one, two and three lengths. For the positive runs, except for BSBR, the longest run has length equal to eight for BBD and ITUB and, again, most of positive runs are concentrated at one, two and three lengths. Moreover, for all banks, there is the same number of positive and negative runs in the considered period. Finally, in Figures 1 and 2, it is presented the frequency plot of the distribution of run lengths as well the histogram of the absolute returns for each bank. From those plots, it could be seen that the run lengths and the absolute returns behaviors have form close to the Weibull distribution. That is, the Weibull (discrete and continuous) distribution could be suitable for the data analysis. Moreover, it is also observed one of the stylized facts of financial markets: the distribution of the daily returns and the distribution of the absolute run returns display heavy tails and sharp peaks.

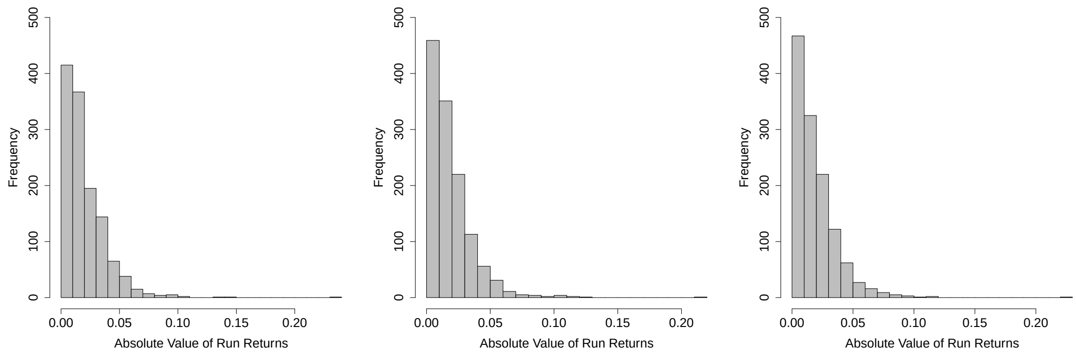

Figure 2: Histogram of absolute returns of historical price stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 (BBD $\rightarrow$ ITUB).

2.3 Zero-Truncated Discrete Weibull Distribution for Run Length

In this subsection, it is assumed that the distribution of the run lengths follows a discrete Weibull (DW) distribution introduced by Nakagawa and Osaki (1975) which can be considered as a discrete analogue of the continuous Weibull distribution. The probability mass function (p.m.f.) of a DW distribution is defined by,

$$\Pr(T = t \mid \phi, \beta) = \phi^{t^\beta} - \phi^{(t+1)^\beta}, \qquad t \in \mathbb{N}_0 = \{0, 1, 2, \ldots\} \tag{2}$$

and its corresponding cumulative distribution function (c.d.f.) is given by,

$$F(t \mid \phi, \beta) = \Pr(T \leq t \mid \phi, \beta) = 1 - \phi^{t^\beta} \tag{3}$$

where $\beta > 0$ and $0 < \phi < 1$. Note that, when $\beta = 1$, the DW distribution reduces to the geometric distribution and when $\beta = 2$, it reduces to the discrete Rayleigh distribution introduced by Roy (2004). This model has been applied to many areas, including competing risks, extreme values, failure times, regional analyses of precipitation, and reliability (see, for example, Khan et al., 1989; Kulasekera, 1994; Roy, 2002; Murthy et al., 2004; Englehardt and Li, 2011; Almalki and Nadarajah, 2014; Brunello and Nakano, 2015). However, the DW cannot be applied directly to the run lengths since the minimum value of a run is one. In this case, a zero-truncation transformation of DW is more appropriate for the data. Thus, let T be a discrete random variable such that $T \sim DW(\phi, \beta)$. The pmf of the zero-truncated discrete Weibull (ZTDW) distribution is defined by,

$$\Pr(T = t \mid \phi, \beta) = \frac{\phi^{t^\beta} - \phi^{(t+1)^\beta}}{\phi}, \qquad t \in \mathbb{N} = \{1, 2, \ldots\} \tag{4}$$

and its corresponding cumulative distribution function (c.d.f.) is given by,

$$F(t \mid \phi, \beta) = \Pr(T \leq t \mid \phi, \beta) = \frac{1 - \phi^{(t+1)^\beta}}{\phi} \tag{5}$$

where $\beta > 0$ and $0 < \phi < 1$.

Now, considering $\boldsymbol{t} = (x_1, t_2, \ldots, t_n)$ as a random sample of size n from the ZTDW distribution, the log-likelihood can be written as,

$$\ell(\beta, \phi \mid \boldsymbol{t}) = \sum_{i=1}^{n} \log\left[\phi^{\boldsymbol{t}^{\beta}} - \phi^{(\boldsymbol{t}+1)^{\beta}}\right] - n\log(\phi) \tag{6}$$

Observe that the log-likelihood expressed in (6) has no compact form which implies that the MLEs and the observed information Fisher's matrix should be obtained using standard numeric optimization algorithms such the Newton-Raphson or the Nelder-Mead methods. However, in this study, inferences for the parameters are based on Bayesian methods obtained using MCMC (Markov Chain Monte Carlo) methods (see Gelfand and Smith, 1990; Chib and Greenberg, 1995). In this way, under a Bayesian approach based on the squared error loss function, $L(\eta, a) = (\eta - a)^2$, it is assumed beta prior distribution for the parameter ϕ since it is restricted to the interval $(0, 1)$ and a uniform prior distribution for the parameter β. The prior distributions are given, respectively, by,

$$\begin{aligned} \pi(\phi) &= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\phi^{(\alpha-1)}(1-\phi)^{\beta-1} \\ \pi(\beta) &\propto 1 \end{aligned} \tag{7}$$

The Bayes estimate of any function of (ϕ, β), say $\omega(\phi, \beta)$ assuming the squared error loss function is given by,

$$\widehat{\mu}_B = \frac{\int_0^1\int_0^\infty \omega(\phi,\beta)L(\phi,\beta)\pi(\phi)\pi(\beta)\,d\phi d\beta}{\int_0^1\int_0^\infty L(\phi,\beta)\pi(\phi)\pi(\beta)\,d\phi d\beta} \tag{8}$$

Since it is not possible to compute Equation (8) analytically, it is used MCMC methods to get the posterior summaries of interest. In this way, without loss of generality, it is considered the Gibbs sampling algorithm to generate samples from the posterior distribution and then compute the Monte Carlo Bayes estimators under the squared error loss function. The Gibbs sampling algorithm steps are given by,

- Step 1: Choose initial values, $\phi^{(0)}$ and $\beta^{(0)}$ for ϕ and β. Denote the values of ϕ and β at the i^{th} step by $\phi^{(i)}, \beta^{(i)}$.
- Step 2: Generate $\phi^{(i)}$, $\beta^{(i+1)}$ from the conditional posterior distributions needed for the Gibbs sampling algorithm obtained directly from the joint posterior distribution.
- Step 3: Repeat step 2, N times.
- Step 4: Calculate the Monte Carlo Bayes estimate of $\omega(\phi, \beta)$ using the expression given by,
$$\frac{1}{N-B}\sum\nolimits_{i=B+1}^{N}\omega(\phi^{(i)}, \beta^{(i)})$$
where $B = 5,000$ is the burn-in period.

The posterior summaries of interest are computed using the package **R2jags** (Su and Yajima, 2012) from R software (R Core Team, 2015) considering a "burn-in sample" of size 5,000 to eliminate the effect of the initial values and a final Gibbs sample of size 2,000 taking every 100th sample from 200,000 simulated Gibbs samples. Furthermore, the convergence of the Gibbs Sampling algorithm was monitored using standard graphical methods, as the trace plots of the simulated samples. Since we are using the statistical package R2jags in the R software we only need the likelihood function and the prior distributions for each parameter of the model and we do not need to specify all conditional posterior distributions required for the Gibbs-Metropolis-Hastings algorithms (the computer codes are available under request to the authors).

2.4 Weibull Distribution for Absolute Returns

In the previous subsection, it was considered the ZTDW distribution as the distribution of the run lengths by the fact that the data was discrete and zero-truncated. However, considering the absolute returns, the ZTDW model is not suitable since the data now is continuous and positive. In this case, it is assumed that the distribution of the absolute returns follows a Weibull (W) distribution. The probability density function (p.d.f.) of a W distribution is defined by,

$$f\left(x \mid \phi, \beta\right) = \frac{\beta}{\phi}\left(\frac{x}{\phi}\right)^{\beta-1} \exp\left\{-\left(\frac{x}{\phi}\right)^{\beta}\right\}, \qquad x \in \mathbb{R}_{+} \tag{9}$$

and its corresponding cumulative distribution function (c.d.f.) is given by,

$$F(x \mid \phi, \beta) = 1 - \exp\left\{-\left(\frac{x}{\phi}\right)^{\beta}\right\} \tag{10}$$

where $\beta > 0$ and $0 < \phi < 1$. For this distribution, when $\beta = 1$, the W distribution reduces to the exponential distribution and when $\beta = 2$, it reduces to the Rayleigh distribution. Different to the ZTDW model, for the W model, it is considered a Classical approach, that is, the parameters here were estimated by the maximum likelihood method and the model fit was assessed by the empiric versus estimated plots of the absolute returns. The log-likelihood for a random sample, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$, of size n of the Weibull can be written as,

$$\ell(\beta, \phi \mid \boldsymbol{x}) = n \log\left(\frac{\beta}{\phi}\right) + (\beta - 1)\sum_{i=1}^{n} \log\left(\frac{\boldsymbol{x}}{\phi}\right) - \beta \sum_{i=1}^{n} \frac{\boldsymbol{x}}{\phi} \tag{11}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$. Observe that the log-likelihood expressed in (11) as the ZTDW model has no compact form which implies that the MLEs and the observed information Fisher's matrix should be also obtained using standard numeric optimization algorithms such the Newton-Raphson or the Nelder-Mead methods. In this paper, the estimation was done by the *fitdistplus* library of the R software.

3 Results and Discussion

In this section, it is presented the analysis of the distributions of run lengths and absolute run returns using the Weibull models presented in Section 2 for each bank in the period from July, 19 2013 to July, 19 2018. Naturally, before fitting Weibull models, the hypothesis that the length of run lengths are independent and identically distributed must be verified. Those assumptions are evaluated from the statistical value of Bartels (Bartels, 1982) and Mann-Kendall tests (Mann, 1945).

According to the Bartels test, in all banks, the i.i.d length hypothesis was not rejected at a significance level of 5%. The hypothesis also passed in the Mann-Kendall test. The p-values associated with the statistics values of both tests were calculated from B = 100,000 permutations (Good, 2006) of the original lengths. The libraries *lawstat* and *Kendall* of the R software were used in the Bartels and Mann-Kendall tests, respectively.

Table 3: Posterior summaries of interest for the positive (negative) runs of BBD, BSBR and ITUB in the period considered assuming the ZTDW model.

		Positive Runs		
Bank	Parameters	Posterior Mean	Std. Dev.	95% Cred. Int.
BBD	ϕ	0.438	0.051	(0.318, 0.513)
	β	0.915	0.069	(0.754, 0.997)
BSBR	ϕ	0.445	0.045	(0.334, 0.511)
	β	0.928	0.060	(0.772, 0.997)
ITUB	ϕ	0.449	0.045	(0.344, 0.517)
	β	0.929	0.059	(0.786, 0.998)
		Negative Runs		
Bank	Parameters	Posterior Mean	Std. Dev.	95% Cred. Int.
BBD	ϕ	0.439	0.055	(0.307, 0.520)
	β	0.904	0.073	(0.725, 0.996)
BSBR	ϕ	0.423	0.049	(0.302, 0.495)
	β	0.919	0.066	(0.756, 0.997)
ITUB	ϕ	0.432	0.059	(0.294, 0.518)
	β	0.898	0.079	(0.710, 0.996)

In the first approach, the run lengths, the parameters of the ZTDW model were estimated by the Bayesian method previously described for positive and negative runs for each bank. The posterior summaries of interest are presented in Table 3. To asses model performance, it is considered the probability plots for the empiric distribution versus the ZTDW distribution as well the traceplot plots for MCMC convergence in each case. The plots are presented in Figures 3, 4 and 5. In terms of the computational cost, the CPU time for user and system was measure in a Core i5-8600K (3.60 Ghz) machine with 16 GB DDR4 RAM and Windows 11 Pro (version 10.0.22621) as operating system. The results showed that the mean of time elapsed for user was 32.68 seconds while the mean of the time elapsed by the system was 0.96 seconds, indicating a great performance for the proposed model.

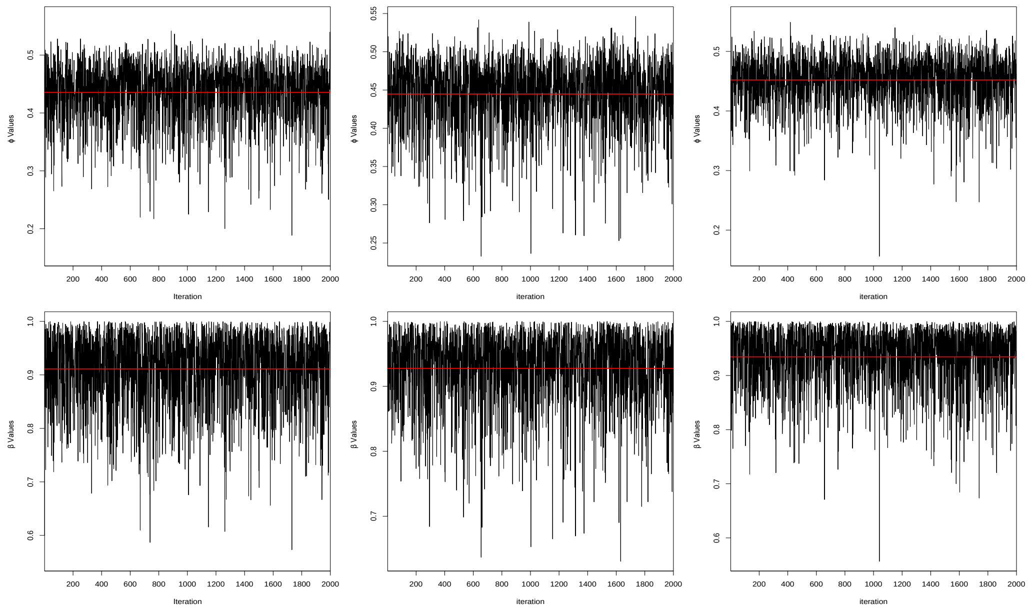

Figure 3: Traceplots for the adjusted ZTDW model for the positive run lengths distribution of historical price stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 (BBD $\rightarrow$ BSBR $\rightarrow$ ITUB. Upper panels: ϕ values; lower panels: β values).

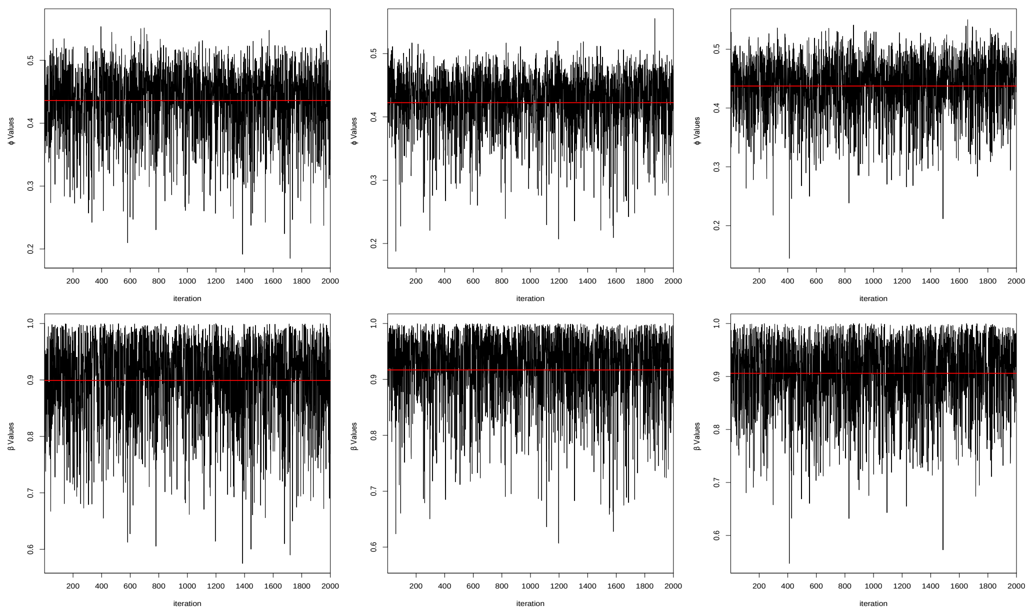

Figure 4: Traceplots for the adjusted ZTDW model for the negative run lengths distribution of historical price stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 (BBD $\rightarrow$ BSBR $\rightarrow$ ITUB. Upper panels: ϕ values; lower panels: β values).

From the results presented in Table 3 and Figure 5, it could concluded that the ZTDW model fits well for the distribution of run lengths with a great accuracy. Moreover, considering a simple random process with two equally likely outcomes such that the pmf of run length should follow an ZTDW distribution, it is possible to conclude that the market is equally likely to go up or go down everyday, ignoring the magnitudes and considering only the signs of the daily returns.

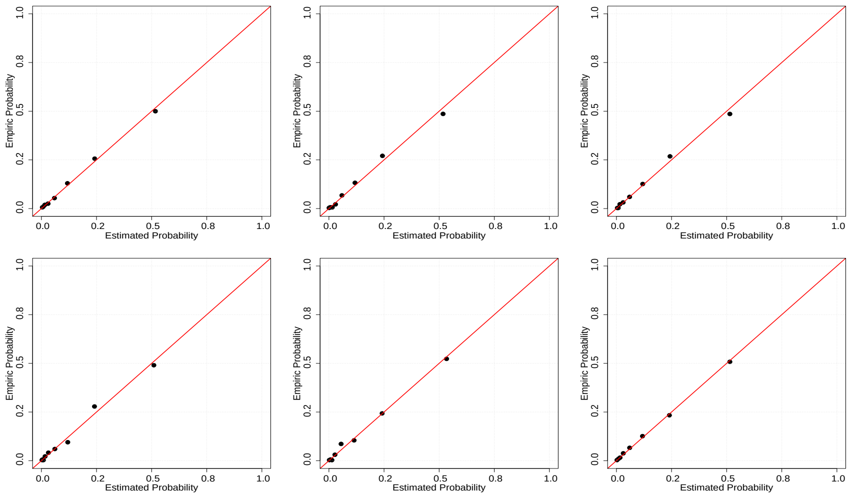

Figure 5: Probability plots for the adjusted ZTDW model for the positive (upper panels) and negative (lower panels) run lengths distribution of historical price stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 (BBD $\rightarrow$ BSBR $\rightarrow$ ITUB).

In the second approach, the absolute returns, the parameters of the W model were estimated by the maximum likelihood method for the absolute returns for each bank. The model summaries of interest are presented in Table 4. To asses model performance, it is considered the plots for the empiric distribution versus the W distribution. The plots are presented in Figure 6 for BBD, Figure 7 for BSBR and Figure 8 for ITUB. In the same way of the run lengths analysis, it is concluded that the W model fits well for the distribution of the absolute returns with a great accuracy which implies that the distribution of the absolute returns is well described using the Weibull model.

Table 4: Maximum likelihood estimates for the absolute returns of BBD, BSBR and ITUB in the period considered assuming the W model.

Bank	Parameters	MLE	Std. Err.	95% Conf. Int.
BBD	ϕ	0.022	0.001	(0.021, 0.023)
	β	1.198	0.026	(1.148, 1.249)
BSBR	ϕ	0.020	0.001	(0.019, 0.021)
	β	1.192	0.025	(1.141, 1.242)
ITUB	ϕ	0.020	0.001	(0.019, 0.021)
	β	1.173	0.026	(1.123, 1.223)

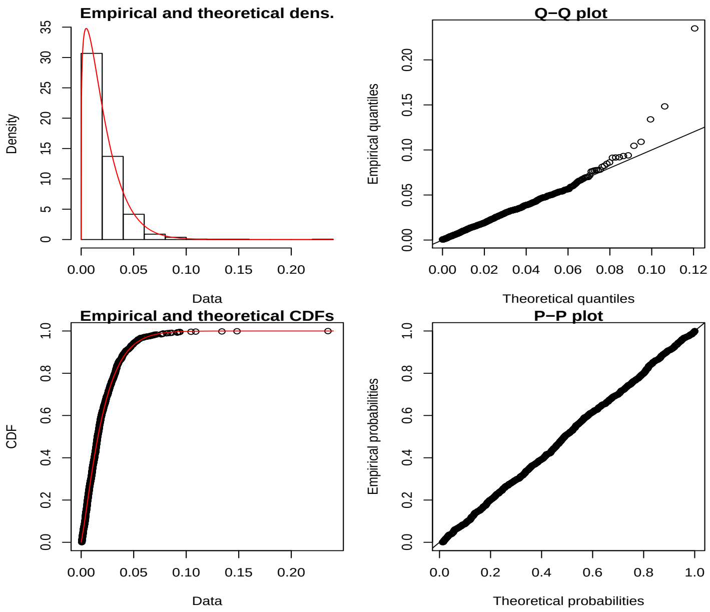

Figure 6: Plots of fitted W model for the absolute returns distribution of historical prices stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 for BBD bank.

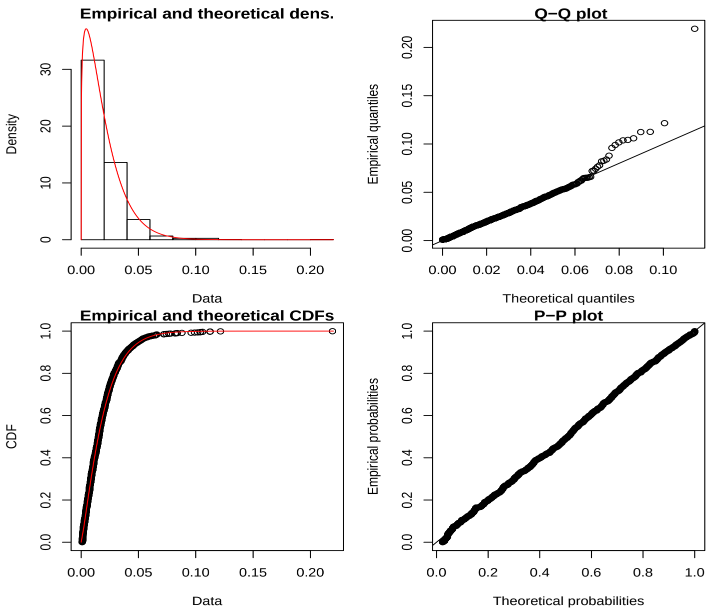

Figure 7: Plots of fitted W model for the absolute returns distribution of historical prices stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 for BSBR bank.

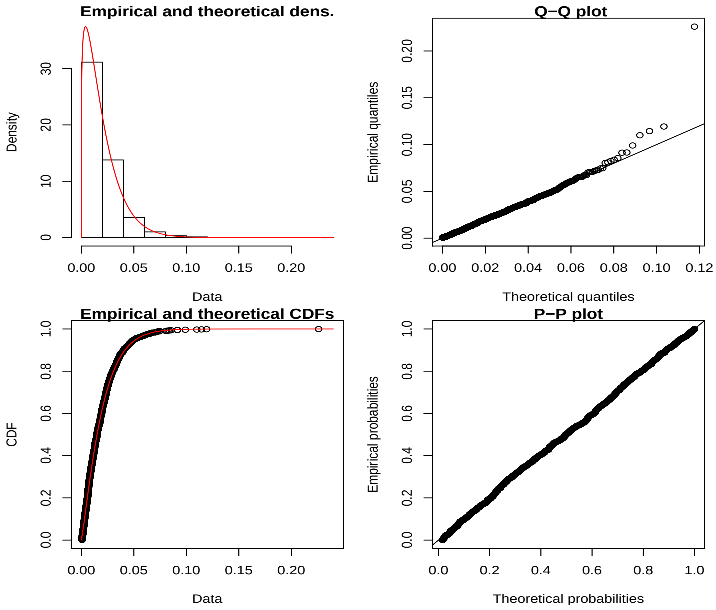

Figure 8: Plots of fitted W model for the absolute returns distribution of historical prices stated in NYSE stock exchange from 19 of July of 2013 until 19 of July of 2018 for ITUB bank.

4 Concluding Remarks

In this paper, it is conducted a detailed analysis on runs of daily returns of three private banks located in Brazil. As a result, it is observed that using a Bayesian approach based on simulation MCMC methodology to get samples of the joint posterior distribution of interest, we could get accurate inferences for the parameters of the ZTDW model even considering very non-informative prior distributions. The model choice provided a great fit the distribution of both length of the runs as well stated that the market is equally likely to go up or go down everyday. For the return absolute value, it could be seen that the Weibull model showed almost a perfect fit for the data, that is, the absolute returns should be Weibull distributed. Once both Weibull models considered here showed great accuracy, the results provided here could be of great importance in financial frequency analysis since the parametric models could provide a way to predict the stock exchange close or open values day-by-day behavior. Also, the major advantage of the proposed model is inherit most of properties of the continuous Weibull model as the asymmetric shape which is the main shape of the distribution of run and return data, and the simplicity of the likelihood function which is a great advantage to get the inferences of interest, since we can use regular algorithms to get the estimators.

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