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The riskiness of longevity indexed life annuities in a stochastic Solvency II perspective

Giovanna Di Lorenzo[a], Albina Orlando*[b], and Massimiliano Politano[c]

[a,c]*Dipartimento di Scienze Economiche e Statistiche, Universita di Napoli Federico II, Via Cinthia Monte Sant'angelo 80131, Napoli*
[b]*Istituto per le Applicazioni del calcolo "Mauro Picone" - Consiglio Nazionale delle Ricerche, Via P. Castellino 111, 80131 Napoli*

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This paper investigates the problem of quantifying the impact of unexpected deviations of mortality trend on a longevity indexed life annuity in a Solvency II perspective. Solvency II quantitative requirements regulate the margins required to offset the insurance risk in a one year risk horizon. Indeed, the idea of deepening the expected changes of future mortality rates over a single year is gaining. In the following the authors propose a computational tractable approach to assess the technical provisions by means of an internal model, in line with Solvency II directives. The impact of adverse effects of the mortality dynamics is investigated. Mortality is modelled by means of a stochastic CIR type model; an ex post analysis is proposed relying on Italian mortality data.

keywords: Longevity indexed life annuities, Solvency II, Techincal provisions, Stochastic mortality models, CIR model.

1 Introduction

Longevity risk is one of the largest risks which insurance companies and annuity providers are exposed to. In the Solvency II directive longevity risk is described as the risk of loss,

*Corresponding author: a.orlando@na.iac.cnr.it

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or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where an adverse change in the mortality rate leads to an increase in the value of insurance liabilities. Hence, the longevity risk is defined as the economic loss stemming from an instantaneous, but permanent, decrease in the mortality intensity used for the calculation of technical provisions. For this reason, actuaries have to employ projected life tables incorporating a forecast of future trends of mortality. Clearly, the risk is that the projections of mortality turn out to be incorrect and the annuitants live longer than expected. Different approaches for the construction of the projected tables have been developed until now, (for a full report on this subject, see Pitacco, 2004), but no one turned out to be suitable for the problem solution. The problem is twofold. On the one hand, insurers have to make the annuity market attractive to the insured. At present, the risk borne out by insurers for insurance annuities, which is undoubtedly too high, is reflected in high premiums charged for these products that discourage individuals who are intending to purchase annuities. On the other hand, Solvency II regulation requires the constitution of appropriate margins that are difficult to bear for an insurance company. The Solvency II directive prescribes a standard formula that an insurance business could use to quantify technical provisions, solvency capital requirements, and market value margins. Solvency II directive allows the use of internal models as an alternative to the standard formula, as long as the internal model follows the Solvency II principles and is approved by regulator.

To solve the problem of making the annuity market more attractive and following the Solvency II requirements, many insurance companies and pension funds providers focus on the issue of sharing the longevity risk. An ordinary way to solve this problem is through reinsurance, but this method often involves high costs. The securitization provides a viable alternative (see Denuit et al., 2007), but unfortunately the longevity bonds are not a very attractive business for investors. Denuit et al. (2011) propose longevity indexed life annuities, an index-type annuity contract where benefits are adjusted over time according to the observed value of a given longevity index.

Richards et al. (2014) observe that there are some circumstancies when it is useful to know by how much expectations of future mortality rates might change over a single year. Such an approach lies at the heart of the one year value at risk view of the actuarial liabilities and is in line with the Solvency II regime for insurers in the European Union. We try to develop this concept combining a stochastic model for mortality rates approach with a quantile simulation procedure for the short period survival probabilities in order to quantify the risk of the insurance position. The proposed approach, which combines a stochastic model for the evolution of mortality rates and a quantile analysis for the mortality distribution, can be useful to capture the trend component of longevity and can help to minimize the security loading in order to front the insurer actuarial liability.

We investigate the problem of quantifying the impact of unexpected deviations of mortality trend on a longevity indexed life annuity. In line with Solvency II directives, we propose a computational tractable approach to assess the technical provisions by means of an internal model.

The paper is organized as follows. In section 1 the longevity index and the longevity indexed life annuity are treated. Section 2 describes the specific features of the Solvency

II directive and introduces a coherent internal model for calculating the insurer risk exposure. In section 3 the issue of modeling the uncertainty in future mortality is fronted and a CIR type model for describing the future evolution of hazard rates is described. Section 4 looks for the conditions that allow to quantify the longevity risk via quantile analysis. Section 5 concludes.

2 The longevity index

Let us consider an individual aged x in the calendar year t. The remaining lifetime is indicated by the notation $T_x(t)$. Therefore, the individual will die at age $x+ T_x(t)$ in the calendar year $t+ T_x(t)$. Then $q_x(t) = P(T_x(t) \leq 1)$ is the probability that an individual aged x in the calendar year t dies before reaching the age $x+1$ and $p_x(t) = 1 - q_x(t) = P(T_x(t) > 1)$ is the probability that the same individual reaches the age $x+1$.

Let $p^{mod}_{x+k}(t+k)$ $(k = 0, 1, 2, ..., \omega)$ be the predicted one year survival probability referred to an individual aged x in the calendar year t deducted by some survival model, where ω denotes the ultimate age. Therefore $p^{mod}_{x+k}(t+k)$ is the assumption that is made on the future mortality.

As time passes, the observed values of the one year survival probabilities $p^{obs}_{x+k}(t+k)$ $(k = 0, 1, 2, ..., \omega)$ become available, so that it is possible to compare the values predicted on the basis of a given model with the actual ones, by means of the following ratio:

$$i_{t+k} = \prod_{j=0}^{k-1} \frac{p^{mod}_{x+j}(t+j)}{p^{obs}_{x+j}(t+j)} \tag{1}$$

which can be assessed each future calendar year k.

In line with Denuit et al. (2011), the basic idea is that annual payment due at time k to an individual buying a longevity indexed annuity at age x in calendar year t, is adjusted by the factor (1). Hence, if the contract specifies an annual payment of 1, the annuitant receives a stream of payments $i_{t+1}, i_{t+2}, ...$ as long as he or she survives. In practice, we consider a basic life annuity contract paying one monetary unit of currency at the end of each year as long as the annuitant survives. The single premium is given by:

$$a_x(t) = E\left[\sum_{k=1}^{T_x(t)} 1^{x,k} v(t,k)\right] = \sum_{k=1}^{\omega-x} v(t,k)\,{}_k p_x(t) \tag{2}$$

where $1^{x,k}$ is an indicator which equals one if the individual aging x at time t is alive in the future year k $(k = 1, 2, 3...., \omega - x)$, $v(t, k)$ is the deterministic discounting factor.

At this point, if the observed survival probabilities are higher than predicted ones, the payments due to the insured are reduced accordingly. The random present value of the longevity indexed life annuity, $a^{L.I.}_x(t)$, is given by:

$$a^{L.I.}_x(t) = E\left[\sum_{k=1}^{T_x(t)} 1^{x,k} i_{t+k} v(t,k)\right] = \sum_{k=1}^{\omega-x} v(t,k)\,{}_k p_x(t) i_{t+k} \tag{3}$$

Through a basic life annuity contract, insurer offers policyholders protection against two broad class of risk: biometric risks, such as longevity and mortality risks, and macroeconomic and financial market risks, such as interest rate, inflation, equity market or credit market risk. In this kind of product insurers bear all risk, both systematic and unsystematic or idyosincratic risk. In a pure longevity- linked annuity, by scaling annuity payments by (1) all the systematic longevity risk is beared by the annuitants. For individual retirees the longevity indexed life annuity offers some of the benefits of a conventional annuity at a potential lower price.

3 The Solvency II directive

The Solvency II Directive applies to all EU insurance and reinsurance companies with gross premium income exceeding 5 million or gross technical provisions in excess of 25 million. It became operative from 1 January 2016.

The key objectives of Solvency II were to increase the level of harmonisation of solvency regulation across Europe, to protect policyholders, to introduce Europe-wide capital requirements that are more sensitive (than the previous minimum Solvency I requirements) to the levels of risk being undertaken, and to provide appropriate incentives for good risk management. The first Pillar sets out the minimum capital requirements that firms are required to meet. The Solvency II Directive prescribes a standard formula that an insurance business should use to assess technical provisions (TPs), solvency capital requirements (SCRs) and a market value margin (MVM). The Solvency II Directive allows so-called internal models as an alternative to the standard formula, as long as the internal model follows the Solvency II principles and is approved by the regulator.

At this proposal, a coherent structure is given by the internal model proposed by Munroe et al. (2015). The technical provision TP, that is the insurance liabilities fair value, is the sum of mean loss and the margin discounted present value.

Referring to a single year run off, the TP is:

$$TP = MVM + BEL \tag{4}$$

Where MVM is the present value of the cost of risk capital for calendar years in run off multiplied by the spread, as prescribed by the cost of capital method:

$$MVM = (1+i)^{-1} \cdot s \cdot Var_{99.5\%}(L_1) \tag{5}$$

In (5) the quantity $Var_{99.5\%}(L_1)$ is the percentile of the loss random variable L_1 for the first future calendar year minus the mean loss for the first year. It represents the SCR, that is the capital required for the first calendar year.

In (4), BEL is the best estimate of liabilities. It is the present value-adjusted sum of the means of calendar year loss distributions:

$$BEL = (1+i)^{-0.5} \cdot s \cdot E(L_1) \tag{6}$$

In (5) and (6), s represents the spread, that is the cost of funding 1 euro of risk capital above the risk free rate i.

In this paper, as a starting point of our research, we refer to this model analyzing the case of a single year run off. In particular we suppose that a longevity indexed annuity contract is issued to individuals of the same age, belonging to different cohorts. Then we look at the effects on the technical provision given by (4).

We focus only on the uncertainty coming from the evolution of mortality, making the hypothesis of deterministic discounting rates (see equation (3)). In order to assess the technical provision, we resort to a quantile analysis of mortality distribution.We propose an ex post analysis, relying on the past mortality experience of the italian male population measured in the period 1960-2014.

4 The stochastic mortality model

Let us consider an individual aged x in the calendar year t. If we consider the hazard rate for an individual aged $x+t$ in the calendar year t. $p_x(t)$ is the probability that an individual reaches the age $t+1$. Analogously, ${}_kp_x(t)$ is the probability that an individual aged x in the calendar year t reaches the age $x+k$ in the year $t+k$.

Considering the instantaneous rate of mortality, the hazard rate μ_{x+t},we have:

$$ {}_kp_x(t) = E[e^{-\int_0^k \mu_{x+s}ds}] \tag{7} $$

Many studies have been proposed in the recent literature, among the others see Bhattacharjee (2016) and Oluyede and Yang (2014). We describe the evolution in time of mortality by a widely used stochastic mortality model (see Biffis, 2005, Dahl, 2006), supposing that the force of mortality at time t for an individual aged $x+t$ is given by

$$ d\mu_{x+t} = \kappa(\gamma - \mu_{x+t})dt + \sigma\sqrt{\mu_{x+t}}dB_t \tag{8} $$

Where κ and σ are positive constants, γ is the long term mean and B_t is a Standard Brownian Motion. This model, referred to as the CIR mortality model has the property that the mortality rates are continuous and remain positive. Moreover, for $2\kappa\gamma \geq \sigma^2$ the mortality rates does not reach zero, and the drift factor $\kappa(\gamma - \mu_{x+t})$ ensures the mean reversion of x+t towards the long term meanγ.

For convenience, we now introduce the centered version of the model. Let us consider the shifted $\mu^*_{x+t} = \mu_{x+t} - \gamma$. The process is then centred around γ and the long term mean converges almost everywhere to zero:

$$ d\mu^*_{x+t} = \kappa\mu^*_{x+t}dt + \sigma\sqrt{\mu^*_{x+t} + \gamma}dB_t \tag{9} $$

with initial condition given by the known value of μ_{x+t}. Its solution is given by

$$ E[\mu^*_{x+t}] = e^{-\kappa t}\mu^*_{x+0} \tag{10} $$

$$ cov(\mu^*_{x+t}, \mu^*_{x+s}) = \sigma^2\frac{e^{-\kappa t} - e^{-\kappa(s+t)}}{\kappa}\mu^*_{x+0} + \sigma^2\frac{e^{-\kappa(t-s)} - e^{-\kappa(s+t)}}{2\kappa}\gamma, s \leq t \tag{11} $$

$$ \lim_{t\to\infty} Var[\mu^*_{x+t}] = \frac{\gamma\sigma^2}{2\kappa} \tag{12} $$

4.1 Parameter estimation

Estimating the parameters of the stochastic mortality model requires the discrete representation of the model. To this aim, we refer to the covariance equivalence principle which requires that the expected values and the stationary variances of the continuous and discrete processes to be equal. The discrete model representation is given by the following equation:

$$\mu^*_{x+t} = \phi\mu^*_{x+t-1} + \sigma_a\sqrt{\frac{2\phi}{1+\phi}\mu^*_{x+t-1} + \gamma a_t} \tag{13}$$

where μ^*_{x+t} is the force of mortality centered around the long term mean γ, as described in section 4. ϕ and σ_a are positive constants: ϕ represents a kind of drift rate and σ_a is a diffusion parameter. a_t, for all t, is a standard Normal variable ($N(0,1)$), independent of $a_i|i \leq 1$.

The expected value, the covariance and stationary variance functions of the previous equation are:

$$E[\mu^*_{x+t}] = \phi^t\mu^*_{x+0} \tag{14}$$

$$cov(\mu^*_{x+t}, \mu^*_{x+s}) = 2\phi^t\sigma_a^2\mu^*_{x+0}\frac{1-\phi^s}{1-\phi^2} + \phi^{t-s}\sigma_a^2\gamma\frac{1-\phi^{2s}}{1-\phi^2} \tag{15}$$

$$\lim_{t\to\infty} Var[\mu^*_{x+t}] = \frac{\sigma_a^2\gamma}{1-\phi^2} \tag{16}$$

The estimation procedure starts by finding the value of ϕ that minimizes the residual sum of squares function:

$$RSS = \sum_{t=1}^{N}\frac{(\mu^*_{x+t} - \phi\mu^*_{x+t-1})^2}{\frac{2\phi}{1+\phi}\mu^*_{x+t} + \gamma} \tag{17}$$

The least squares estimate of σ_a^2 is given by RSS/N-1. Finally the continuous model parameters are obtained by means of the parametric relationships between continuous and discrete models, derived by applying the covariance equivalence principle:

$$\phi = e^{-\kappa} \tag{18}$$

$$\sigma_a^2 = \sigma^2\frac{1-e^{-2\kappa}}{2\kappa} \tag{19}$$

At this point, by the Pitman and Yor formula (Pitman and Yor (1982)), we can compute the survival probability

$${}_kp_x(t) = E[e^{-\int_0^k \mu_{x+s}ds}] = \frac{exp(-\frac{x}{\sigma^2}w\frac{1+\frac{k}{w}coth(wk/2)}{coth(wk/2)} + (k/w))}{(coth(wt/2) + (k/w)sinhcoth(wk/2))^{\frac{2k\gamma}{\sigma^2}}} \tag{20}$$

where $x = \mu_0$ e $w = \sqrt{k^2 + s\sigma^2}$

5 Numerical results

In order to assess the technical provision given by (4) we need to quantify the expected value of liabilities $E[L_1]$ and the quantile $Var_{99.5\%}(L_1)$.

We assume that the discounting factor is deteministic and the uncertainty comes from the survival probabilities. Therefore we resort to a stochastic simulation procedure and derive, year by year, a set of cumulative probabilites and we estimate the related quantiles.

5.1 The dataset

Our set of data relates to the Italian male population with annual age-specific death counts ranging from ages 64 to 89 over the period 1954 to 2008 (data source: Human mortality database www.mortality.org). In line with Orlando and Politano (2013), we refer to the class of the forward mortality models. These models study changes in the mortality rate curve for a specific age cohort and capture dynamics of each age cohort over time for all ages greater than x in a specific year t (for example age x in the year t, $x + 1$ in the year $t + 1$ and so on). In this case, the mortality curves are modeled diagonally (for example see Dahl and Moller, 2004, Cairns et al., 2006, Bauer et al., 2009). In practice, on the basis of data available for the previous 25 years, we can estimate the model parameters for the year t and, as a result, it is possible to get the forecasted survival probabilities. For example, with the data of the period 1954-1978 it is possible to obtain the column of the survival probabilities for the year 1979. This procedure is repeated thirty times in order to obtain the annual survival probabilities over the period 1979 to 2008 and ranging from ages 64 to 89. These probabilities can be compared with the corresponding survival rates obtained from the tables of the Human Mortality Database. Regarding the choice of fixing the extreme age to 89, recent studies (Khalaf et al., 2006) have shown that the most damaging effects in terms of annuities present values for the provider are in the age range 73-80. Clearly this happens because the number of survival is still large at these ages. As a consequence, even modest improvements in the level of survival probabilities with respect to those used for pricing and reserving, result in large additional costs for the annuity provider. We estimate the parameters, following the procedure described in section (4.1). We choose to calculate the long term mean as the simple mean of each historical series used to estimate the parameters. takes the same value for each calendar year. The reason can be found in the high autoregressive parameter of the discrete model ϕ=0.999 , which is the same each year explaining the high correlation of each data of each series with the preceding one.

5.2 The quantile analysis and the technical provision assessment

At this point we model the future uncertainty about mortality by means of the CIR type stochastic process. In practice, the longevity index (1) is computed as:

$$i_{t+k}^{CIR} = \prod_{j=0}^{k-1} \frac{p_{x+j}^{CIR}(t+j)}{p_{x+j}^{obs}(t+j)} \tag{21}$$

where:
$p_{x+j}^{CIR}(t+j)$ is a forecast of the annual survival probability of a male aged 64 in 1983. The forecasted probabilities are obtained by means of the CIR type stochastic process on the basis of the parameters estimated; $p_{x+j}^{obs}(t+j)$ are the actual values of the annual survival probabilities deducted from the Italian male mortality tables for the period 1983- 2008. In formula (21), $p_{x+j}^{CIR}(t+j)$ are calculated by means of (20), using the estimated parameters for the year 1983, based on the mortality experience of the years 1958-1982.

Now, on the basis of mortality data for the last 25 years, the model is able to provide a good fit to the real survival probabilites of the next year but, unfortunately, fails in projection. In other word, it is not able to capture the decrease in time of the parameters and γ because of the well known phenomena of rectangularization and expansion of the Lexis point.

The quantile estimation gives an important information to the insurer by quantifying the tail events. In our analysis we refer to a tail event as the event of a survival probability higher than the expected one. We consider the survival probabilities derived by the stochastic model described in the previous section. Fixing the age x we resort to a stochastic simulation procedure and derive, in a one year horizon, a set of cumulative probabilities. Then we estimate the related quantiles. We simulate a large number N of sample paths, each of one producing a simulated set of ${}_tp_x^s(S=1,,N)$. We set N=10000. The mortality risk measure we refer to is the quantile $MRM = q_\alpha$ where α is the confidence level chosen, in our case the 99.5 per cent. In order to perform the simulation procedure it is necessary to consider the discrete time equation for the chosen Stochastic Differential Equation describing the evolution in time of the mortality rates. On the basis of the first order Euler discretization of equation (8), with a time interval $[x+t, x+t+1]$ we have:

$$\mu_{x+t_k+1} = \mu_{x+t_k} + \kappa(\lambda - \mu_{x+t_k})\Delta + \sigma\sqrt{\mu_{x+t_k}\Delta}\epsilon_k, k = 1, 2, 3, ..., n-1 \tag{22}$$

where$\Delta = 1/n$ is the sampling interval , with ϵ_k being the increment ΔB_k of the Wiener process between $t_k = k\Delta$ and $t_{k+1} = (k+1)\Delta$. The increments ΔB_k are $N(0,\Delta)$ distributed random variables. The discretized process is then represented by the sequence $[\mu_{t_1}, \mu_{t_2}, ..., \mu_{t_n}]$. By means of (7), we obtain the corresponding survival probabilities.

For each simulated set, we study the longevity index given by (21) given by the ratio between the simulated survival probabilities and the real data.

Basing on our study, the response of the model is good in the medium term. For all the years considered the results of the reports are quite close to 1 up to age 78. The probability of underestimating the survival probabilities is negligible. The situation is different for age greater than 78. In this case the probability of underestimating grows up to 24%. Around this level the probability of underestimation stabilizes even for higher age.

These comments can be extended to the other years of the dataset. Finally, we suppose that a longevity indexed annuity contract is issued to individuals of the same age, 65, belonging to different cohorts. Then we look at the effects on the technical provision given by (4).

In particular, we are interested in the effects on the technical provision due to changes in the mortality rate curve for a specific age cohort x in a specific year t, taking into account the mortality dynamics over time. In terms of technical provision for our

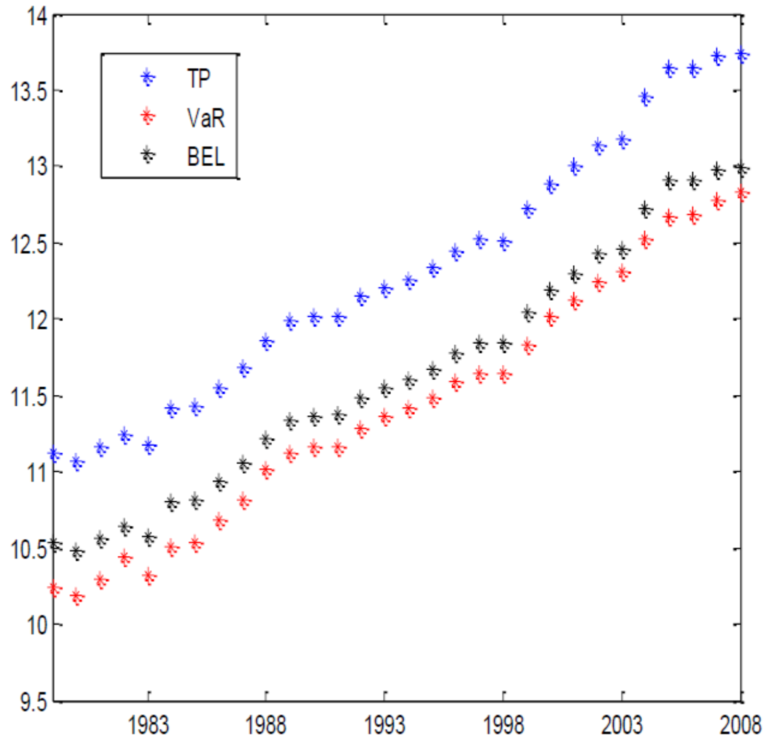

Figure 1: Technical provision. VAR 99.5%. Best Estimate Liabilities

longevity indexed life annuity, the increase achieved by effect of longevity is at about 23 with respect to the total actuarial liability. This result remains almost constant throughout the period considered is obviously dependent on the insured individuals over the age of 80. At this point it is possible to draw two conclusions. The first is that the combination between a stochastic model for the evolution of the force of mortality and a quantile approach allows to control the deviations of mortality from its expected trend due to the longevity and to limit the probability of underestimation within precise limits. So in our case an increase of the 23% reserve, limits to 0.5% the probability of incurring in losses. Clearly, if the reserves are used for prudential purposes, benefits for the insureds and remuneration for the shareholders are limited.

The second is that the introduction of a single threshold to describe uncontrolled deviations of mortality from its trend looks wrong. The composition of the funds by the insurance company should better fit its risk profile.

Indeed, we observe that the effect of longevity is evident in the advanced ages and then it depends effectively on the age classes of the insured.

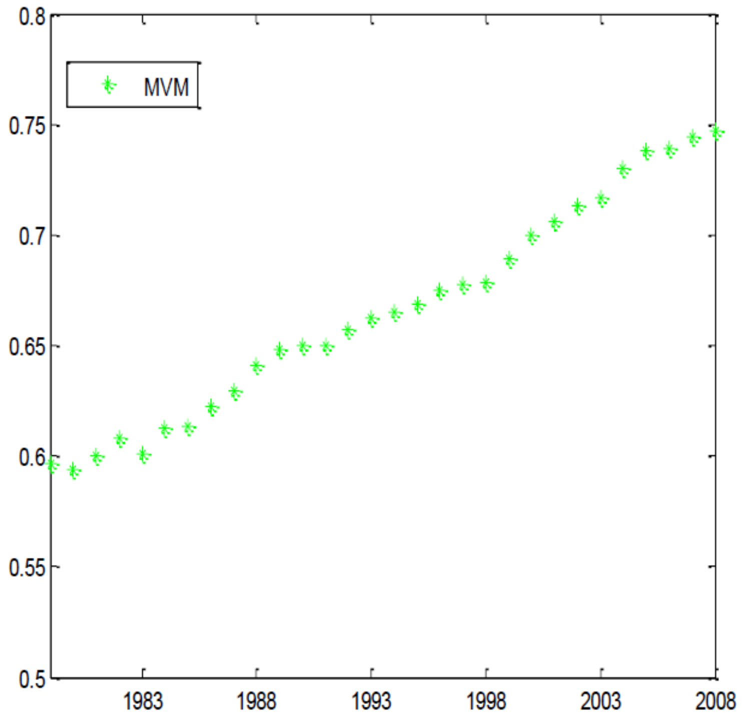

Figure 2: Market value margin

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