Electronic Journal of Applied Statistical Analysis
EJASA, Electron. J. App. Stat. Anal.
http://siba-ese.unisalento.it/index.php/ejasa/index
e-ISSN: 2070-5948
DOI: 10.1285/i20705948v10n1p257

Some Bayes estimators for Pareto Type-II progressive censored data

By Prakash

Published: 26 April 2017

Electronic Journal of Applied Statistical Analysis
Vol. 10, Issue 01, 2017, 257-270
DOI: 10.1285/i20705948v10n1p257

Some Bayes estimators for Pareto Type-II progressive censored data

Gyan Prakash*

Department of Community Medicine
M. L. N. Medical College, Allahabad, U. P., India

Published: 26 April 2017

Two-parameter Pareto distribution is not only a potential model for life testing problems, also established its important role in variety of other problems such as size of cities, firms and business mortality. In present article, two-parameter Pareto distribution is considered as the underlying model for the study. Some Bayes estimators are obtained for shape parameter under known as well as unknown case of scale parameter. A Progressive Type-II censored data is considered and performances of the procedures are studied in terms of relative efficiency, based on simulated and a real life data.

keywords: Pareto Type-II distribution, Bayes estimator, Invariant LINEX loss function, Progressive Type-II censoring.

1 Introduction

The Pareto distribution plays an important role in analysing areas including city population distribution, stock price fluctuation, oil field locations and military areas. The decreasing failure rate of present distribution, provides a useful model survival after some medical procedures. Its close relatives provide a very flexible family of fat-tailed distributions, which may be applicable for income distribution of higher income group.

The Pareto distribution has established its grate role in variety of other problems discussed by several authors time to time. For example, Lomax (1954) discussed its applicability in business mortality, Harries (1967) service time in queuing system, size of cities and firms by Steindle (1965) and the study of distributions of nuclear particles by

*Corresponding author: ggyanji@yahoo.com

ISSN: 2070-5948
http://siba-ese.unisalento.it/index.php/ejasa/index

Freiling (1966).

A lots of literature are available on Pareto model under classical and Bayesian methodology, few resents works are discussed herewith. Recently, Prakash (2014 a) obtained some Bayes estimators for Lomax model under right ordered sample data. Some Statistical inference for generalized Pareto distribution based on Progressive Type-II censored data with random removals was discussed by Azimi et al. (2014). Okasha (2014) concerned in his article about the using of E-Bayesian method for computing estimates of the unknown parameter, reliability and hazard functions of Lomax distribution based on type-II censored data.

Al-Zahrani and Al-Sobhi (2013) was discussed estimation problem of the probability $S = P(Y < X)$ for Lomax distribution based on general progressive censored data. The maximum likelihood estimator and Bayes estimators are also obtained using the symmetric and asymmetric balanced loss functions. Some Bayes prediction bound length of intervals for Pareto model has studied by Prakash and Singh (2013). Fu at al. (2012) discussed about the objective Bayesian analysis of Pareto distribution under progressive Type-II censoring. Three different types of non-informative priors and two general forms of second order probability matching prior are used. Li (2011) derived ML and Bayes estimates for the shape parameter, reliability, and failure rate functions of the Pareto distribution under progressive Type-II censored samples.

In this paper, Bayesian analysis for the shape parameter of Pareto Type-II distributions under Progressive Type-II censoring is considered. Two different loss functions have used for a comparative analysis based on Relative efficiency obtained for Bayes estimator when scale parameter is considered to be known as well as unknown. A simulated and real life data are both analyzed for the purpose of illustration.

2 The Model and Posterior Densities

The probability density function of Pareto Type-II distribution with shape parameter θ and scale parameter σ is given as

$$f(x;\theta,\sigma) = \theta\sigma^{\theta}(x+\sigma)^{-\theta-1} \;\; ; \;\; x \geq 0, \theta > 0, \sigma > 0. \tag{1}$$

Progressive censoring is more flexible than the ordinary censoring, has now one of the most popular censoring schemes in life testing. In recent literature about progressive censoring, both classical and subjective Bayesian approaches are widely utilized in parameter estimation of various distributions. For a comprehensive recent review of progressive censoring, see Balakrishnan (2007).

Let us take an experiment with n independent and identical units $X_1, X_2, ..., X_n$ at beginning time and first $m; (1 \leq m \leq n)$ failure times are observed. At time of each failure occurring prior to termination point, one or more surviving units are removed from the test. The experiment is terminated at the time of m^{th} failure, and all remaining surviving units are removed from the test (See Prakash (2015) for more detail)

Based on progressively Type-II censoring scheme the joint probability density function of order statistics $X_{1:m:n}^{(R_1,R_2,\ldots,R_m)}, X_{2:m:n}^{(R_1,R_2,\ldots,R_m)}, \ldots, X_{m:m:n}^{(R_1,R_2,\ldots,R_m)}$ is defined as

$$f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}\left(\theta,\sigma|\underline{x}\right) = k_m \prod_{i=1}^{m} f\left(x_{(i)};\theta,\sigma\right)\left(1-F\left(x_{(i)};\theta,\sigma\right)\right)^{R_i}. \tag{2}$$

Here, k_m be the Progressive normalizing constant. Simplifying Eq.(2), the joint probability density function is obtained as:

$$f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}\left(\theta,\sigma|\underline{x}\right) = k_m \prod_{i=1}^{m}\left(\theta\sigma^{\theta}\left(x_{(i)}+\sigma\right)^{-\theta-1}\right)\left(\frac{\sigma}{x_{(i)}+\sigma}\right)^{\theta R_i}$$

$$\Rightarrow f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}\left(\theta,\sigma|\underline{x}\right) = k_m\theta^m\, e^{-\theta T_m(\underline{x};\theta,\sigma)}e^{-T_0(\underline{x};\sigma)}; \tag{3}$$

where $T_0\left(\underline{x};\sigma\right) = \sum_{i=1}^{m} log\left(x_{(i)}+\sigma\right)$ and $T_m\left(\underline{x};\theta,\sigma\right) = \sum_{i=1}^{m}\left(R_i+1\right) log\left(x_{(i)}+\sigma\right) - n$ $log\sigma$.

The two-parameter Gamma distribution is considered as the conjugate family of prior density for the shape parameter θ when scale parameter σ is known. The probability density function of Gamma density is given as

$$\pi(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}\,;\theta>0,\alpha>0,\beta>0. \tag{4}$$

Now, the posterior density for the parameter θ is defined as

$$\pi^*\left(\theta|\underline{x}\right) = \frac{f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}\left(\theta,\sigma|\underline{x}\right)\cdot\pi(\theta)}{\int_{\theta} f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}\left(\theta,\sigma|\underline{x}\right)\cdot\pi(\theta)d\theta} \tag{5}$$

Using Eq. (3) & Eq. (4) in Eq. (5) we get

$$\pi^*\left(\theta|\underline{x}\right) \propto \frac{\theta^m\, exp\left(-\theta T_m\left(\underline{x};\theta,\sigma\right) - T_0\left(\underline{x};\sigma\right)\right)\cdot\theta^{\alpha-1}e^{-\beta\theta}}{\int_{\theta}\theta^m\, exp\left(-\theta T_m\left(\underline{x};\theta,\sigma\right) - T_0\left(\underline{x};\sigma\right)\right)\cdot\theta^{\alpha-1}e^{-\beta\theta}d\theta}$$

$$\Rightarrow \pi^*\left(\theta|\underline{x}\right) = \frac{\left(T_m\left(\underline{x};\theta,\sigma\right)+\beta\right)^{m+\alpha}}{\Gamma(m+\alpha)}\,\theta^{m+\alpha-1}\, e^{-\theta\left(T_m(\underline{x};\theta,\sigma)+\beta\right)}. \tag{6}$$

When scale parameter σ is consider as a random variable, the joint prior density is thus defined as

$$\pi_1\left(\theta,\sigma\right) = \pi_1^*\left(\theta|\sigma\right)\cdot\pi_1^*(\sigma).$$

Following Prakash (2014 b), the joint prior density for the parameters θ and σ is thus obtained as

$$\pi_1(\theta,\sigma) = \frac{\gamma^{\beta}}{\Gamma(\beta)\Gamma(\alpha)}\theta^{\alpha-1}e^{-\sigma\theta}\sigma^{\alpha+\beta-1}e^{-\gamma\sigma}. \tag{7}$$

Using Eq.(7) in Eq. (5), the joint posterior density when both parameters is considered to be unknown is obtained as

$$\pi^{**}(\theta,\sigma|\underline{x}) = \frac{f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}(\theta,\sigma|\underline{x})\cdot\pi_1(\theta,\sigma)}{\int_\theta\int_\sigma f_{(X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n})}(\theta,\sigma|\underline{x})\cdot\pi_1(\theta,\sigma)d\sigma d\theta}$$

$$= \frac{\theta^{m+\alpha-1}exp\left(-\theta\left(T_m(\underline{x};\theta,\sigma)+\sigma\right)\right)exp\left(-T_0(\underline{x};\sigma)-\gamma\sigma\right)\sigma^{\alpha+\beta-1}}{\int_\sigma exp\left(-T_0(\underline{x};\sigma)-\gamma\sigma\right)\sigma^{\alpha+\beta-1}\left(\int_\theta \theta^{m+\alpha-1}exp\left(-\theta\left(T_m(\underline{x};\theta,\sigma)+\sigma\right)\right)d\theta\right)d\sigma}$$

$$\Rightarrow \pi^{**}(\theta,\sigma|\underline{x}) = \frac{\theta^{m+\alpha-1}e^{-\theta\hat{T}_m(\underline{x};\theta,\sigma)}\,e^{-\hat{T}_0(\underline{x};\sigma)}\sigma^{\alpha+\beta-1}}{\Gamma(m+\alpha)\bar{\sigma}} \tag{8}$$

where $\hat{T}_m(\underline{x};\theta,\sigma) = T_m(\underline{x};\theta,\sigma)+\sigma$, $\bar{\sigma} = \int_\sigma \frac{e^{-\hat{T}_0(\underline{x};\sigma)}\sigma^{\alpha+\beta-1}}{\hat{T}_m(\underline{x};\theta,\sigma)^{\alpha+m}}d\sigma$ and $\hat{T}_0(\underline{x};\sigma) = T_0(\underline{x};\sigma)$ $+\gamma\sigma$.

3 Bayes Estimator Under Squared Error Loss (SELF)

The choice of loss function may be crucial in Bayesian analysis. The most commonly used loss function is a squared error loss function. The Bayes estimator for parameter θ corresponding to posterior density $\pi^*(\theta|\underline{x})$ under SELF is obtained as

$$\hat{\theta}_{S1} = \frac{\left(T_m(\underline{x};\theta,\sigma)+\beta\right)^{m+\alpha}}{\Gamma(m+\alpha)}\int_\theta \theta^{m+\alpha}e^{-\theta(T_m(\underline{x};\theta,\sigma)+\beta)}d\theta$$

$$\Rightarrow \hat{\theta}_{S1} = \frac{m+\alpha}{T_m(\underline{x};\theta,\sigma)+\beta}. \tag{9}$$

Similarly, the Bayes estimator corresponding to parameter θ for posterior density $\pi^{**}(\theta,\sigma|\underline{x})$ is obtained as

$$\hat{\theta}_{S2} = \int_\sigma\int_\theta \theta\frac{\theta^{m+\alpha-1}e^{-\theta\hat{T}_m(\underline{x};\theta,\sigma)}e^{-\hat{T}_0(\underline{x};\sigma)}\sigma^{\alpha+\beta-1}}{\Gamma(m+\alpha)\bar{\sigma}}\,d\theta\,d\sigma$$

$$\Rightarrow \hat{\theta}_{S2} = (m+\alpha)\left(\frac{\bar{\bar{\sigma}}}{\bar{\sigma}}\right)\ ;\ \bar{\bar{\sigma}} = \int_\sigma \frac{e^{-\hat{T}_0(\underline{x};\sigma)}\sigma^{\alpha+\beta-1}}{\left(\hat{T}_m(\underline{x};\theta,\sigma)\right)^{m+\alpha+1}}d\sigma. \tag{10}$$

A nice close form of the Bayes estimator $\hat{\theta}_{S2}$ is not exist, however a numerical finding have been presented. The expression of the risks corresponding to SELF is defined as

$$R_{(S)}\left(\hat{\theta}_{Si}; i = 1, 2\right) = E_{T_{(\underline{x})}}\left(\hat{\theta}^2_{Si}\right) - 2\theta E_{T_{(\underline{x})}}\left(\hat{\theta}_{Si}\right) + \theta^2.$$

Again close forms of the expressions of the risk do not exist. A numerical technique with help of simulation is applied herewith for drawing relative efficiency for Bayes estimators corresponding to different loss functions.

4 Bayes Estimator Under Invariant Linex Loss (ILLF)

When overestimation is more serious than underestimation, or vice-versa, or positive and negative errors have different consequences, the use of SELF is not appropriate in estimation problems. An invariant LINEX loss function (ILLF) is useful and flexible (See Singh at al. (2007) for more details) and is given as

$$L(\Delta) = e^{a\partial} - a\partial - 1\ ;\ \partial = \frac{\hat{\theta}}{\theta} - 1.$$

Here, $'a'$ is called as the shape parameter and $\hat{\theta}$ be the any estimate of the parameter θ.

The Bayes estimator $\hat{\theta}_{L1}$ corresponding to ILLF under posterior density $\pi^*(\theta|\underline{x})$ is obtain by simplify the following equality

$$E\left(\theta e^{a\hat{\theta}_{L1}\theta}\right) = e^a E(\theta)$$

$$\Rightarrow \int_\theta \theta^{m+\alpha}\, e^{-\theta\left(T_m(\underline{x};\theta,\sigma)+\beta-a\hat{\theta}_{L1}\right)} d\theta = e^a \int_\theta \theta^{m+\alpha}\, e^{-\theta(T_m(\underline{x};\theta,\sigma)+\beta)} d\theta$$

$$\Rightarrow \hat{\theta}_{L1} = \frac{a^*}{a}\left(T_m(\underline{x};\theta,\sigma) + \beta\right)\ ;\ a^* = \left(1 - e^{-a/(m+\alpha-1)}\right). \qquad (11)$$

where $T_m(\underline{x};\theta,\sigma) = \sum_{i=1}^m (R_i + 1)\, log\left(x_{(i)} + \sigma\right) - n\ log\sigma.$

Similarly, Bayes estimator $\hat{\theta}_{L2}$ corresponding to posterior density $\pi^{**}(\theta, \sigma|\underline{x})$ is obtained as

$$E\left(\theta e^{a\hat{\theta}_{L2}\theta}\right) = e^a E(\theta)$$

$$\Rightarrow \int_\sigma \int_\theta \frac{\theta^{m+\alpha}\, e^{-\theta\left(\hat{T}_m(\underline{x};\theta,\sigma)-a\hat{\theta}_{L2}\right)}\, e^{-\hat{T}_0(\underline{x};\sigma)}\, \sigma^{\alpha+\beta-1}}{\bar{\sigma}} d\theta d\sigma$$

$$= e^{a} \int_{\sigma} \int_{\theta} \frac{\theta^{m+\alpha} \; e^{-\theta\left(\hat{T}_m(\underline{x};\theta,\sigma)\right)} \; e^{-\hat{T}_0(\underline{x};\sigma)} \; \sigma^{\alpha+\beta-1}}{\bar{\sigma}} d\theta d\sigma$$

$$\Rightarrow \int_{\sigma} \frac{e^{-\hat{T}_0(\underline{x};\sigma)} \; \sigma^{\alpha+\beta-1}}{\left(\hat{T}_m\left(\underline{x};\theta,\sigma\right) - a\hat{\theta}_{L2}\right)^{m+\alpha+1}} d\sigma = e^{a} \int_{\sigma} \frac{e^{-\hat{T}_0(\underline{x};\sigma)} \; \sigma^{\alpha+\beta-1}}{\left(\hat{T}_m\left(\underline{x};\theta,\sigma\right)\right)^{m+\alpha+1}} d\sigma \qquad (12)$$

The risk corresponding to the Bayes estimators under ILLF is

$$R_{(L)}\left(\hat{\theta}_{Li}; i=1,2\right) = e^{-a} E_{T(\underline{x})}\left(exp\left(a\frac{\hat{\theta}_{Li}}{\theta}\right)\right) - aE_{T_{(\underline{x})}}\left(\frac{\hat{\theta}_{Li}}{\theta}\right) + a - 1.$$

Again, the closes forms of the risks do not exist. A numerical integration method has been applied here for obtaining their risks.

5 Numerical Analysis

The close form of the risk expressions under both loss functions for all Bayes estimators do not exists. A numerical integration techniques based on the simulation is applied here for studying the properties of Bayes estimators. Now, the relative efficiency under both risks criterion are defined as

$$RE_{(S)}\left(\hat{\theta}_{Li}, \hat{\theta}_{Si}; i=1,2\right) = \frac{RE_{(S)}\left(\hat{\theta}_{Si}\right)}{RE_{(S)}\left(\hat{\theta}_{Li}\right)}$$

and

$$RE_{(L)}\left(\hat{\theta}_{Li}, \hat{\theta}_{Si}; i=1,2\right) = \frac{RE_{(L)}\left(\hat{\theta}_{Si}\right)}{RE_{(L)}\left(\hat{\theta}_{Li}\right)}.$$

CASE 1. When Scale Parameter Is Known

To assess and study the properties of the Bayes estimator for shape parameter θ, a simulation study has been performed. The random samples are generated as follows:

1. Generate θ through prior density $\pi(\theta)$ for selected set of prior parameters α and β as $(\alpha, \beta) = (0.25, 0.50), (4, 2), (9, 3)$. The selections of prior parametric values meet the criterion that the prior variance should be unity.

2. Using the values of shape parameter θ obtain from step (1) with considered set of values of scale parameter $\sigma = 0.50(0.50)2.50$; generate $10,000$ random samples of size $N = 30$ from model (1).

Table 1: Censoring Scheme for Different Values of m

$Case$	m	$R_i \; \forall \, i = 1, 2, ..., m$
1	10	1 0 0 3 0 0 1 0 0 1
2	20	1 0 2 0 0 1 0 0 1 0 1 0 1 1 0 1 0 2 0 0

3. The progressively Type-II censored Pareto Type-II data is generated, by the help of known σ and generated θ of size m, under a given censoring plan $R_i; i = 1, 2, ..., m$, according to an algorithm proposed by Balakrishnan and Aggarwala (2000).
4. The progressively Type-II censoring plan for different values of m is presented in Table 1.
5. The Relative efficiencies between $\hat{\theta}_{S1}$ & $\hat{\theta}_{L1}$ under SELF & ILLF have been obtain and presented in Tables 2-3 respectively.
6. The RE's under both risk criterion are presented here for $a = 0.25, 0.50, 1.00$ at $N = 30$.
7. From both the tables, it is observed that the Bayes estimator $\hat{\theta}_{L1}$ performs uniformly better than the estimator $\hat{\theta}_{S1}$. The Relative efficiencies are higher for small scale parameter. The RE decreases when $'a'$ is increases.
8. The Relative efficiency further decrease when set of prior parameter is upgraded. Similar trend also has seen when progressive censoring pattern and size are changed.

Case 2. When Scale Parameter Unknown

When both parameters are considered as the random variable, a simulation study also has been carried out for studying the properties of Bayes estimators as:

1. The scale parameter σ has been generated for set of values of prior parameter β and γ chosen as $(\beta, \gamma) = (0.25, 0.50), (4, 2)$ and $(9, 3)$ and obtained from the prior distribution
$$\pi'(\sigma) = \frac{\gamma^{\beta}}{\Gamma(\beta)} e^{-\gamma\sigma} \sigma^{\beta-1}.$$
The selection criterion for the prior parametric values is similar.
2. Using generate values of σ from Step (1) and selected values of $\alpha = 0.50(0.50)2.50$; the shape parameter θ has been generated from the prior
$$\pi''(\theta|\sigma) = \frac{\sigma^{\alpha}}{\Gamma(\alpha)} e^{-\sigma\theta} \theta^{\alpha-1}.$$

Table 2: Relative Efficiency Between $\hat{\theta}_{S1}$ and $\hat{\theta}_{L1}$ Under SELF

$N = 30$			$\leftarrow \sigma \rightarrow$				
m	a	$(\alpha, \beta) \downarrow$	0.50	1.00	1.50	2.00	2.50
		0.25, 0.50	2.2068	2.1526	2.0540	1.9188	1.7565
	0.25	04, 02	1.7218	1.6795	1.6026	1.4971	1.3705
		09, 03	1.6292	1.5892	1.5164	1.4166	1.2967
		0.25, 0.50	1.7583	1.7151	1.6365	1.5288	1.3995
10	0.50	04, 02	1.2988	1.2698	1.2088	1.1293	1.0337
		09, 03	1.2819	1.2504	1.1932	1.1147	1.0204
		0.25, 0.50	1.6796	1.6383	1.5633	1.4604	1.3369
	1.00	04, 02	1.2864	1.2548	1.1973	1.1185	1.0239
		09, 03	1.2783	1.1718	1.1036	1.0376	1.0185
		0.25, 0.50	1.9234	1.6202	1.5477	1.4800	1.3492
	0.25	04, 02	1.7217	1.6172	1.5409	1.3929	1.1378
		09, 03	1.5877	1.5791	1.4608	1.3672	1.0923
		0.25, 0.50	1.4229	1.3866	1.3635	1.2538	1.2432
20	0.50	04, 02	1.2705	1.2676	1.1964	1.1147	1.0325
		09, 03	1.2598	1.2501	1.1598	1.1104	1.0152
		0.25, 0.50	1.2278	1.2262	1.2189	1.1413	1.0619
	1.00	04, 02	1.2233	1.1954	1.1953	1.0898	1.0195
		09, 03	1.2194	1.1608	1.0975	1.0203	1.0142

Table 3: Relative Efficiency Between $\hat{\theta}_{S1}$ and $\hat{\theta}_{L1}$ Under ILLF

$N = 30$			$\leftarrow \sigma \rightarrow$				
m	a	$(\alpha, \beta) \downarrow$	0.50	1.00	1.50	2.00	2.50
		0.25, 0.50	3.9529	3.6039	3.4343	3.1326	2.9414
	0.25	04, 02	2.8828	2.8813	2.8108	2.5704	2.5036
		09, 03	2.7251	2.7211	2.5904	2.3755	2.3714
		0.25, 0.50	2.9547	2.8724	2.7496	2.5624	2.3499
10	0.50	04, 02	2.7496	2.6404	2.4251	2.1835	1.8319
		09, 03	2.4666	2.3973	2.2813	2.1766	1.8187
		0.25, 0.50	2.8158	2.7439	2.6179	2.4429	2.2301
	1.00	04, 02	2.5429	2.4856	2.2499	2.1823	1.7714
		09, 03	2.4063	2.3962	2.1848	2.1737	1.7705
		0.25, 0.50	3.2218	2.7187	2.5973	2.4705	2.2576
	0.25	04, 02	2.8754	2.7081	2.5881	2.2576	1.9053
		09, 03	2.6582	2.6577	2.4428	2.2513	1.8916
		0.25, 0.50	2.3855	2.2951	2.2844	2.0961	2.0802
20	0.50	04, 02	2.2986	2.1976	2.1489	1.9667	1.7234
		09, 03	2.1768	2.1294	2.1288	1.8594	1.7059
		0.25, 0.50	2.0514	2.0421	1.9411	1.9117	1.7782
	1.00	04, 02	2.0357	2.0144	1.9142	1.8249	1.7227
		09, 03	2.0048	1.9434	1.8378	1.8059	1.6973

Table 4: Relative Efficiency Between $\hat{\theta}_{S2}$ and $\hat{\theta}_{L2}$ Under SELF

$N = 30$			$\leftarrow \sigma \rightarrow$				
m	a	$(\beta, \gamma) \downarrow$	0.50	1.00	1.50	2.00	2.50
		0.25, 0.50	3.0253	2.7541	2.6284	2.5132	2.1198
	0.25	04, 02	2.7268	2.6252	2.5143	1.9872	1.1128
		09, 03	2.3653	2.2591	1.9856	1.8807	1.1049
		0.25, 0.50	2.6177	2.3898	2.3404	1.9613	1.7984
10	0.50	04, 02	2.4404	2.2082	1.9808	1.8623	1.4365
		09, 03	2.1666	2.0629	1.9297	1.6714	1.2043
		0.25, 0.50	2.5507	2.1001	2.0036	1.8696	1.7685
	1.00	04, 02	1.9606	1.9231	1.7259	1.6421	1.3739
		09, 03	1.8469	1.8329	1.7214	1.6369	1.3505
		0.25, 0.50	2.6501	2.5754	2.3841	2.0795	1.8529
	0.25	04, 02	2.2685	2.1722	1.9808	1.9785	1.8219
		09, 03	2.2441	2.1348	1.9595	1.8733	1.7734
		0.25, 0.50	2.2254	2.1756	2.0836	1.8604	1.5929
20	0.50	04, 02	2.1759	2.1683	1.9648	1.8505	1.5192
		09, 03	1.8781	1.8341	1.7491	1.6589	1.1914
		0.25, 0.50	1.8737	1.5621	1.4856	1.3631	1.3409
	1.00	04, 02	1.8582	1.5479	1.4531	1.3601	1.3187
		09, 03	1.5343	1.4879	1.4358	1.3214	1.2997

3. Using other Steps as discussed in previous section, the Relative efficiencies between $\hat{\theta}_{S2}$ & $\hat{\theta}_{L2}$ under SELF & ILLF have been obtain and presented in Tables 4-5 respectively.

4. From both the tables, it is observed again that the Bayes estimator $\hat{\theta}_{L2}$ performs uniformly better than the estimator $\hat{\theta}_{S2}$. Other properties have been seen similar as discussed above.

CASE 3. A Real Life Example

In present section the properties of Bayes estimation are studied under an example uses real-life data. Lawless (1982) considered data representing the break-down times (in minutes) of an insulating fluid between electrodes at a voltage of 34 kV. Total 18 observations are given in Table 6. A Progressively Type-II censored sample scheme of

Table 5: Relative Efficiency Between $\hat{\theta}_{S2}$ and $\hat{\theta}_{L2}$ Under ILLF

$N = 30$			$\leftarrow \sigma \rightarrow$				
m	a	$(\beta, \gamma) \downarrow$	0.50	1.00	1.50	2.00	2.50
		0.25, 0.50	4.2791	3.7229	3.6994	3.1921	2.9718
	0.25	04, 02	3.9375	3.6359	3.2641	3.1277	2.9511
		09, 03	3.7768	3.3727	3.2637	2.8429	2.6464
		0.25, 0.50	3.1077	2.9608	2.8178	2.6125	2.5449
10	0.50	04, 02	2.8778	2.6956	2.6712	2.4935	2.2663
		09, 03	2.5408	2.5402	2.3491	2.2204	1.8533
		0.25, 0.50	2.8692	2.7997	2.6759	2.4258	2.2242
	1.00	04, 02	2.5115	2.4412	2.3262	2.2149	1.8015
		09, 03	2.4563	2.3276	2.2905	2.2112	1.6409
		0.25, 0.50	3.2829	2.9701	2.9468	2.7582	2.3441
	0.25	04, 02	2.9965	2.7549	2.6723	2.4402	1.9456
		09, 03	2.7843	2.7134	2.5956	2.4041	1.9296
		0.25, 0.50	2.3076	2.2353	2.1879	2.0877	1.9675
20	0.50	04, 02	2.2419	2.2103	2.1397	2.0041	1.7604
		09, 03	2.1818	2.1609	2.1275	1.9085	1.7549
		0.25, 0.50	2.0032	1.9808	1.9775	1.9477	1.8944
	1.00	04, 02	1.9431	1.9026	1.8505	1.7553	1.6755
		09, 03	1.4244	1.4079	1.3725	1.3417	1.2909

Table 6: The Break-Down Times of An Insulating Fluid Between Electrodes

0.19	0.78	0.96	1.31	2.78	3.16
4.15	4.67	4.85	6.50	7.35	8.01
8.27	12.06	31.75	32.52	33.91	36.71

size $m = 10$ is selected randomly. The relative efficiency for all the Bayes estimators under considered risk functions are obtained and presented in the Table 7. The relative efficiency are obtained for $N = 18$ and $a = 0.50$ only. All the properties are seen similar as discussed above.

6 Conclusions

Pareto Type-II distribution is considered here as the underlying model for study the properties of Bayes estimator under different loss function. A Progressive Type-II censored data has been utilised under the SELF and ILLF risk criterion. For known and unknown both case of scale parameter is taken for obtaining the Bayes estimators for shape parameter. Since, the close form of the risks under both risks criterion does not exist, so a numerical technique under a simulation has been carried out for study the performances of the procedures. We observe that the Bayes estimator under ILLF is perform better as compare to Bayes estimator under SELF in both cases.

Table 7: Relative Efficiency Between $\hat{\theta}_{S1}$ and $\hat{\theta}_{L1}$ Under SELF

Relative Efficiency Between $\hat{\theta}_{S1}$ and $\hat{\theta}_{L1}$ Under SELF					
	$\leftarrow \sigma \rightarrow$				
$(\alpha, \beta) \downarrow$	0.50	1.00	1.50	2.00	2.50
0.25, 0.50	2.5915	2.2501	2.2115	2.0137	1.8284
04, 02	1.8119	1.8089	1.7428	1.6852	1.6114
09, 03	1.7766	1.7304	1.6783	1.6074	1.4547
Relative Efficiency Between $\hat{\theta}_{S1}$ and $\hat{\theta}_{L1}$ Under ILLF					
0.25, 0.50	3.1061	3.0769	2.9506	2.7135	2.6914
04, 02	3.0007	2.8681	2.8189	2.6698	2.1314
09, 03	2.9294	2.7074	2.5115	2.4166	1.9435
Relative Efficiency Between $\hat{\theta}_{S2}$ and $\hat{\theta}_{L2}$ Under SELF					
	$\leftarrow \alpha \rightarrow$				
$(\beta, \gamma) \downarrow$	0.50	1.00	1.50	2.00	2.50
0.25, 0.50	2.7847	2.4279	2.3964	2.2639	2.0647
04, 02	2.5947	2.3438	2.1728	2.0109	1.9316
09, 03	2.2091	2.0594	1.9034	1.7273	1.5632
Relative Efficiency Between $\hat{\theta}_{S2}$ and $\hat{\theta}_{L2}$ Under ILLF					
0.25, 0.50	3.0332	3.0247	3.0014	2.8498	2.6283
04, 02	2.8303	2.8008	2.5528	2.4072	2.1814
09, 03	2.4607	2.3439	2.1526	2.1501	1.9979

References

Al-Zahrani, B. & Al-Sobhi, M. (2013). On parameters estimation of Lomax distribution under general Progressive censoring. *Journal of Quality and Reliability Engineering*, 2013, 1-7.

Azimi, R., Fasihi, B., & Sarikhanbaglu, F. A. (2014). Statistical inference for generalized Pareto distribution based on Progressive Type-II censored data with random removals. *International Journal of Scientific World*, 2 (1), 1-9.

Balakrishnan, N., (2007). Progressive censoring methodology: an appraisal. *Mathematics and Statistics*, 16, 211 - 259.

Balakrishnan, N. & Aggarwala, R. (2000). Progressive Censoring: Theory, Methods and Applications. Birkhauser Publishers, Boston.

Freiling, E. C. (1966). A comparison of the fallout mass-size distributions calculated by lognormal and power-law models. U.S. Naval Radiological Defence Laboratory, San Francisco.

Fu, J., Xu, A. & Tang, Y. (2012). Objective Bayesian analysis of Pareto distribution under Progressive Type-II censoring. *Statistics and Probability Letters*, 82, 1829 - 1836.

Harries, C. M. (1967). Queues with stochastic service rate. *Naval Research Logistics Quarterly*, 14, 219 - 230.

Lawless J. F. (1982). Statistical models and methods for lifetime data New York, Wiley.

Li, F. (2011). Reliability Analysis for the Pareto Model Based on the Progressive Type II Censored Sample. *Advances in Computer Science, Intelligent System, and Environment*, 104, 467 - 472.

Lomax, K. S. (1954). Business failures. Another example of the analysis of failure data. *Journal of the American Statistical Association*, 49, 847 - 852.

Okasha, H. M. (2014). E-Bayesian estimation for the Lomax distribution based on type-II censored data. *Journal of the Egyptian Mathematical Society*, 22 (3), 489 - 495.

Prakash, G. (2014 a). Right censored Bayes estimator for Lomax model. *Statistics Research Letters*, 3 (1), 23 - 28.

Prakash, G. (2014 b). Bayes prediction bounds for right ordered Pareto TypeII data. *Journal of Statistics Applications & Probability*, 3 (3), 335 - 343.

Prakash, G. (2015). A comparative study based on Bayes estimation under different censoring criterion. *Journal of Data Science*, 13 (2), 261 - 280.

Prakash, G. and Singh, D. C. (2013). Bayes prediction intervals for the Pareto model. *Journal of Probability and Statistical Science*, 11(1), 109 - 122.

Singh, D. C., Prakash, G. and Singh, P. (2007). Shrinkage testimators for the shape parameter of Pareto distribution using LINEX loss function. *Communications in Statistics - Theory and Methods*, 36, 741 - 753.

Steindle, J (1965). Random processes and the growth of firms, a study of the Pareto law. New York, Heffner.