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Some aging properties of Weibull models

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In this paper, we derive the reversed hazard rate of some well-known Weibull models, which are widely used in reliability analysis. The comparison of reversed hazard rate along with hazard rate, and aging intensity function is done with the help of numerical examples.

keywords: Hazard rate, aging intensity function, reversed hazard rate, Weibull models.

1 Introduction

In the context of reliability theory, some well-known functions are available, viz., survival function, hazard rate, reversed hazard rate, mean residual to study lifetime distributions or statistical data. The notations used throughout the paper are mentioned in sequel. We denote a continuous lifetime random variable by X with probability density function $f(\cdot)$, cumulative distribution function $F(\cdot)$, survival function $\bar{F}(\cdot)$, hazard rate function $r(\cdot) = f(\cdot)/\bar{F}(\cdot)$, reversed hazard rate function $\mu(\cdot) = f(\cdot)/F(\cdot)$. $r(t)$ is widely used in aging analysis of a device, whereas the importance of $\mu(t)$ is found in the forensic Science, where exact time of failure (*i.e.*, death in case of living beings) of a system is of importance. One can refer to Barlow and Proschan (1981), Shanthikumar and Shaked (1994), and Shaked and Shanthikumar (2007) among others. Recently, the role of aging intensity function (AI) in analyzing aging phenomenon quantitatively is significantly discussed by Jiang et al. (2003), Nanda et al. (2007), and Bhattacharjee et al. (2013). AI function $L(\cdot)$ is defined as $L(t) = r(t)/H(t)$ with

$$H(t) = \frac{1}{t}\Big(\int_0^t r(u)du\Big)$$

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where defined. It can be seen that

$$L(t) = \frac{-t\ f(t)}{\bar{F}(t)\ln \bar{F}(t)},\ \text{for}\ > 0$$

We present a short summary of some generalized Weibull models to provide a better fitting of certain data sets than other available distributions. Some of the families of recent Weibull models have been highlighted in Nadarajah and Kotz (2005) and Pham and Lai (2007).

Survival Function		Notation used
$\bar{F}_X(t) = \exp\left(-\frac{t^\alpha}{\beta}\right),\ \alpha, \beta > 0, t \geq 0$	:	$X \sim W_2(\alpha, \beta)$ (Weibull, 1951)
$\bar{F}_X(t) = \exp\left(-at^\beta e^{bt}\right),\ a > 0, \beta > 0, b \geq 0, t \geq 0$	:	$X \sim W_3(a, \beta, b)$ (cf. Lai et al., 2003)
$\bar{F}_X(t) = 1 - \exp\left\{-(\frac{\beta}{t})^\alpha\right\}, \alpha, \beta > 0, t \geq 0$	:	$X \sim W_I(\beta, \alpha)$ (cf. Jiang et al., 2001)
$\bar{F}_X(t) = \exp\left[-\lambda\left(\frac{t-a}{b-t}\right)^\beta\right],\ 0 \leqslant a < t < b; \lambda, \beta > 0$	:	$X \sim W_4(a, b, \lambda, \beta)$ (Kies, 1958)
$\bar{F}_X(t) = \exp\left[1 - a^{t^\alpha}\right],\ \alpha, a > 0, t \geq 0$	:	$X \sim W_P(a, \alpha)$ (cf. Pham, 2002)
$\bar{F}_X(t) = \exp\left[\frac{\theta}{\alpha}\left(1 - \exp(\alpha t)\right)\right],\ t \geqslant 0,\ \theta > 0,\ \alpha \in \mathbb{R}$	:	$X \sim W_G(\theta, \alpha)$ (cf. Gompertz, 1825)
$\bar{F}_X(t) = \exp\left\{1 - e^{(\lambda t)^\alpha}\right\}\ \alpha, \lambda > 0, t \geq 0,$	:	$X \sim W_S(\lambda, \alpha)$ (cf. Smith and Bain, 1975)

* As reported in Pham and Lai (2007).

In Section 2, we derive the reversed hazard rate of some well-known Weibull models, which are widely used in reliability analysis. The comparison of reversed hazard rate with hazard rate, and aging intensity function is done with the help of numerical examples in Section 3. The paper ends with concluding remarks in Section 4.

2 Reversed hazard rate function

We derive the reversed hazard rate function of a family of Weibull distributions listed in Section 1.

Theorem 2.1 *If $X \sim W_2(\alpha, \beta)$, then its reversed hazard rate is a decreasing function of t.*

Proof. If $X \sim W_2(\alpha, \beta)$, then

$$f_X(t) = \exp\Big(-\frac{t^\alpha}{\beta}\Big)\frac{\alpha t^{\alpha-1}}{\beta}$$

so that

$$\mu_X(t) = \frac{\alpha}{\beta} t^{\alpha-1} \left(\frac{1}{\exp\left(\frac{t^\alpha}{\beta}\right) - 1} \right)$$

Thus,

$$\begin{aligned} \frac{d}{dt}\mu_X(t) &= \frac{-t^{\alpha-2}\alpha\Big\{(\alpha-1)\beta + \exp\left(\frac{t^\alpha}{\beta}\right)(t^\alpha\alpha + \beta - \alpha\beta)\Big\}}{\beta^2\Big\{\exp\left(\frac{t^\alpha}{\beta}\right) - 1\Big\}^2} \\ &= \frac{-t^{\alpha-2}\alpha}{\beta^2\Big\{\exp\left(\frac{t^\alpha}{\beta}\right) - 1\Big\}^2} A(t) \qquad (1) \end{aligned}$$

where

$$A(t) = (\alpha-1)\beta + \exp\left(\frac{t^\alpha}{\beta}\right)(t^\alpha\alpha + \beta - \alpha\beta)$$

Note that

$$\frac{d}{dt}A(t) = \frac{\alpha}{\beta} t^{\alpha-1} \exp\left(\frac{t^\alpha}{\beta}\right)\Big(t^\alpha\alpha + \beta\Big)$$

which is non-negative for all $t \geq 0$. Thus, $A(t)$ is increasing in t. Also, $A(0) = 0$. Since, A(t) is increasing in t, it follows that $A(t) \geq A(0)$ for $t \geq 0$, giving rise to $A(t) \geq 0$ for all $t \geq 0$. Thus in (1), we find that

$$\frac{d}{dt}\mu_X(t) \leq 0, \quad \text{for all} \quad t \geq 0.$$

Thus, for two-parameter Weibull, reversed hazard rate is a decreasing function of t. □

Theorem 2.2 *If $X \sim W_3(a, \beta, b)$, then the reversed hazard rate is a decreasing function of t.*

Proof. Here,

$$f_X(t) = at^{\beta-1}(bt+\beta)e^{bt - a\exp(bt)t^\beta}$$

$$\mu_X(t) = \frac{ae^{bt}t^{\beta-1}(bt+\beta)}{e^{ae^{bt}t^\beta} - 1},$$

so that

$$\frac{d}{dt}(\mu_X(t)) = \frac{W(t)}{\Big(e^{ae^{bt}t^\beta} - 1\Big)^2} \qquad (2)$$

where,

$$\begin{aligned} W(t) &= ae^{bt}t^{\beta-2}\Bigg\{ - ae^{bt+ae^{bt}t^\beta}t^\beta(bt+\beta)^2 + \Big(-1 + e^{ae^{bt}t^\beta}\Big)\Big\{-\beta + (bt+\beta)^2\Big\}\Bigg\} \\ &= ae^{bt}t^{\beta-2}\Bigg[(bt+\beta)^2\Big\{-ae^{bt+ae^{bt}t^\beta}t^\beta - 1 + e^{ae^{bt}t^\beta}\Big\} - \beta\Big\{-1 + e^{ae^{bt}t^\beta}\Big\}\Bigg] \\ &= ae^{bt}t^{\beta-2}\Bigg[(bt+\beta)^2 W_1(t) - \beta\Big\{-1 + e^{ae^{bt}t^\beta}\Big\}\Bigg] \qquad (3) \end{aligned}$$

such that $W_1(t) = \Big\{ -ae^{bt+ae^{bt}t^{\beta}}t^{\beta} - 1 + e^{ae^{bt}t^{\beta}} \Big\}$. Note that,

$$\frac{d}{dt}(W_1(t)) = -a^2 e^{2bt+ae^{bt}t^{\beta}} t^{2\beta-1}(bt+\beta),$$

which is negative for $t \geq 0$. Thus, $W_1(t)$ is decreasing in t. Also, $W_1(0) = 0$, which gives $W_1(t) \leq W_1(0) = 0$. Hence in (3), we find that $W(t) \leq 0$, which leads to the fact that $\mu_X(t)$ in (2) is a decreasing function of t. □

Theorem 2.3 *If $X \sim W_I(\beta, \alpha)$, then $\mu_X(t)$ is a decreasing function of t.*

Proof. Here,

$$f_X(t) = \frac{\alpha}{t}\Big(\frac{\beta}{t}\Big)^{\alpha} e^{-(\frac{\beta}{t})^{\alpha}},$$

$$\mu_X(t) = \frac{\alpha\beta^{\alpha}}{t^{\alpha+1}}$$

for $t > 0$. Thus, $\mu_X(t)$ is a decreasing function of $t > 0$. □

Theorem 2.4 *If $X \sim W_4(a, b, \lambda, \beta)$, then the reversed hazard rate is a decreasing function of t, for $a < t < (a+b)/2$.*

Proof. Here,

$$f_X(t) = \frac{\beta\lambda(a-b)e^{-\lambda(\frac{t-a}{b-t})^{\beta}}\Big(\frac{t-a}{b-t}\Big)^{\beta}}{(a-t)(b-t)},$$

$$\mu_X(t) = \frac{\beta\lambda(a-b)\Big(\frac{t-a}{b-t}\Big)^{\beta}}{(a-t)(b-t)\Big(e^{\lambda(\frac{t-a}{b-t})^{\beta}} - 1\Big)},$$

so that

$$\frac{d}{dt}\Big(\mu_X(t)\Big) = \frac{\beta\lambda(a-b)\Big(\frac{t-a}{b-t}\Big)^{\beta}}{(a-t)^2(b-t)^2\Big(e^{\lambda(\frac{t-a}{b-t})^{\beta}} - 1\Big)^2} W(t) \tag{4}$$

where,

$$\begin{aligned} W(t) &= \Bigg[\Big\{ -1 + e^{\lambda(\frac{t-a}{b-t})^{\beta}} \Big\}(a+b-2t+a\beta-b\beta) - (a-b)\beta\lambda e^{\lambda(\frac{t-a}{b-t})^{\beta}}\Big(\frac{t-a}{b-t}\Big)^{\beta}\Bigg] \\ &= \Bigg[\Big\{ -1 + e^{Y} \Big\}(a+b-2t+a\beta-b\beta) - (a-b)\beta e^{Y} Y\Bigg] \\ &= \beta(a-b)(e^{Y} - 1 - Ye^{Y}) + (e^{Y}-1)(a+b-2t) \\ &= f_1(t) + f_2(t) \end{aligned} \tag{5}$$

where $Y \equiv Y(t) = \lambda\Big(\frac{t-a}{b-t}\Big)^{\beta}$, $f_1(t) = \beta(a-b)(e^{Y} - 1 - Ye^{Y})$, and $f_2(t) = (e^{Y}-1)(a+b-2t)$. It is to be noted that $f_1(t) \geq 0$ for all $a < t < b$, since $(a-b) \leq 0$ and

$(e^Y - 1 - Ye^Y) \leq 0$ for all $t \geq 0$. and $f_2(t) \geq 0$ for $a < t < (a+b)/2$. Thus, from (5), we conclude that $W(t) \geq 0$, so that in (4) we have $\frac{d}{dt}\Big(\mu_X(t)\Big) \leq 0$, for $a < t < (a+b)/2$. Consequently, $\mu_X(t)$ is a decreasing function of t for $a < t < (a+b)/2$. □

Theorem 2.5 *Let $X \sim W_P(a, \alpha)$, then the reversed hazard rate is a decreasing function of t if $\alpha < 1$.*

Proof. Here,

$$f_X(t) = a^{t^\alpha} t^{\alpha-1} \alpha \ln(a) e^{1-a^{t^\alpha}},$$

so that

$$\mu_X(t) = \frac{e a^{t^\alpha} t^{\alpha-1} \alpha \ \ln(a)}{e^{a^{t^\alpha}} - e} \tag{6}$$

Note that,

$$\begin{aligned} \frac{d}{dt}\mu_X(t) &= \frac{a^{t^\alpha} e t^{\alpha-2} \alpha \ln a \left\{ (\alpha-1)(e^{a^{t^\alpha}} - e) - \{e + (a^{t^\alpha} - 1)e^{a^{t^\alpha}}\} t^\alpha \alpha \ln(a) \right\}}{(e - e^{a^{t^\alpha}})^2} \\ &= \frac{a^{t^\alpha} e t^{\alpha-2} \alpha \ln a}{(e - e^{a^{t^\alpha}})^2} W(t) \end{aligned} \tag{7}$$

where

$$W(t) = \left\{ (\alpha-1)(e^{a^{t^\alpha}} - e) - \{e + (a^{t^\alpha} - 1)e^{a^{t^\alpha}}\} t^\alpha \alpha \ln(a) \right\} \tag{8}$$

Let $g(t) = e^{a^{t^\alpha}}$ for $t \geq 0$. Here, $g^{'}(t) = a^{t^\alpha} e^{a^{t^\alpha}} t^{\alpha-1} \alpha \ln a$, so that $g(t)$ is a decreasing (increasing) function of t if $a < (>)1$. Hence, $g(t) \leq (\geq) g(0)$ if $a < (>)1$, i.e., $e^{a^{t^\alpha}} - e \leq (\geq) 0$ if $a <\ (>)1$. Similarly, let $h(t) = a^{t^\alpha}$ for $t \geq 0$. Here, $h^{'}(t) = a^{t^\alpha} t^{\alpha-1} \alpha \ln a$, so that $h(t)$ is a decreasing (increasing) function of t if $a < (>)1$. Hence, $h(t) \leq (\geq) h(0)$ if $a < (>)1$, i.e., $a^{t^\alpha} - 1 \leq (\geq) 0$ if $a < (>)1$.
Let us consider the two cases separately: *Case 1.* $\alpha < 1, a < 1$, and *Case 2.* $\alpha < 1, a > 1$. Let us first study *Case 1.*, in which we find, $e \geq e^{a^{t^\alpha}}$ for $t \geq 0$, so that

$$\begin{aligned} e + \left(a^{t^\alpha} - 1\right) e^{a^{t^\alpha}} &> e^{a^{t^\alpha}} + \left(a^{t^\alpha} - 1\right) e^{a^{t^\alpha}} \\ &\geq e^{a^{t^\alpha}} + \left(a^{t^\alpha} - 1\right) e^{a^{t^\alpha}} \\ &\geq e^{a^{t^\alpha}} (1 + a^{t^\alpha} - 1) \\ &\geq 0. \end{aligned}$$

Hence, $\Big[\Big\{e + (a^{t^\alpha} - 1)e^{a^{t^\alpha}}\Big\} t^\alpha \alpha \ln(a)\Big] \leq 0$. Further, $\left\{(\alpha - 1)(e^{a^{t^\alpha}} - e)\right\} \geq 0$, thus in (8), we conclude $W(t) \geq 0$ for *Case 1.* Using (7), in *Case 1* (i.e., $\alpha < 1, a < 1$), we find $\mu_X(t)$ is a decreasing function of t. In *Case 2*, we note that in (8), $W(t) \leq 0$, since $\Big\{(\alpha-1)(e^{a^{t^\alpha}}-e)\Big\} \leq 0$ and $\Big[\Big\{e+(a^{t^\alpha}-1)e^{a^{t^\alpha}}\Big\}t^\alpha\alpha\ln(a)\Big] \geq 0$. Hence, in *Case 2* (i.e., $\alpha < 1, a > 1$), we find $\mu_X(t)$ is a decreasing function of t. Thus, we conclude that, $\mu_X(t)$ is a decreasing function of t for $\alpha < 1$, and for any $a > 0$. □

Theorem 2.6 *Let $X \sim W_G(\theta,\alpha)$, then the reversed hazard rate is a decreasing function of t if $\alpha \geq 0, \theta > 1$. Further, reversed hazard rate is a decreasing function of t if $\alpha < 0$.*

Proof. Here,

$$f_X(t) = \theta e^{\alpha t+\frac{\theta}{\alpha}(1-e^{\alpha t})}$$

so that

$$\mu_X(t) = \frac{\theta e^{\alpha t+\frac{\theta}{\alpha}}}{e^{\frac{\theta e^{\alpha t}}{\alpha}}-e^{\frac{\theta}{\alpha}}} \tag{9}$$

Note that,

$$\frac{d}{dt}\mu_X(t) = \frac{\theta e^{\alpha t+\frac{\theta}{\alpha}}\Bigg\{-\alpha e^{\frac{\theta}{\alpha}}+e^{\frac{\theta e^{\alpha t}}{\alpha}}(\alpha-\theta e^{\alpha t})\Bigg\}}{\Big\{e^{\frac{\theta e^{\alpha t}}{\alpha}}-e^{\frac{\theta}{\alpha}}\Big\}^2} \tag{10}$$

It is easy to note that if $\alpha \geq 0$, $e^{\alpha t} \geq \alpha$ for $t \geq 0$. Further, $\theta e^{\alpha t} \geq \alpha$ if $\theta > 1$. Therefore, from (10), we conclude $\mu_X(t)$ is a decreasing function of t, if $\alpha \geq 0$ and $\theta > 1$. Next, we proceed to study $\mu_X(t)$ if $\alpha < 0$. Let $G(t) = \Big(e^{\frac{\theta}{\alpha}}-e^{\frac{\theta e^{\alpha t}}{\alpha}}\Big)$. Here, $\frac{d}{dt}G(t) = -\theta e^{\alpha t+\frac{\theta}{\alpha}e^{\alpha t}}$, so that $G(t)$ is a decreasing function of t. Note that, $G(t) \leq G(0)$, for $t \geq 0$, gives rise to $e^{\frac{\theta}{\alpha}} \leq e^{\frac{\theta e^{\alpha t}}{\alpha}}$. Also, $\alpha\Big(e^{\frac{\theta e^{\alpha t}}{\alpha}}-e^{\frac{\theta}{\alpha}}\Big) \leq 0$ if $\alpha < 0$. Thus, in (10), we find $\frac{d}{dt}\mu_X(t) \leq 0$ if $\alpha < 0$. Hence, we conclude that $\mu_X(t)$ is a decreasing function of t if $\alpha < 0$. □

Theorem 2.7 *Let $X \sim W_S(\lambda,\alpha)$, then the reversed hazard rate is a decreasing function of t if $\alpha < 1$.*

Proof. Here,

$$f_X(t) = \frac{\alpha(\lambda t)^\alpha}{t}\Big\{e^{1-e^{(\lambda t)^\alpha}+(\lambda t)^\alpha}\Big\}$$

so that

$$\mu_X(t) = \frac{\alpha(t\lambda)^\alpha e^{1+(\lambda t)^\alpha}}{t\Big\{e^{e^{(\lambda t)^\alpha}}-e\Big\}} \tag{11}$$

Note that,

$$\begin{aligned}\frac{d}{dt}\mu_X(t) &= \frac{\alpha(t\lambda)^\alpha e^{1+(\lambda t)^\alpha}\Bigg\{(\alpha-1)(e^{e^{(\lambda t)^\alpha}}-e)-\{e+(e^{(\lambda t)^\alpha}-1)e^{e^{(\lambda t)^\alpha}}\}\alpha(\lambda t)^\alpha\Bigg\}}{t^2\Big\{e-e^{e^{(\lambda t)^\alpha}}\Big\}^2}\\ &= \frac{\alpha(t\lambda)^\alpha e^{1+(\lambda t)^\alpha}}{t^2\Big\{e-e^{e^{(\lambda t)^\alpha}}\Big\}^2}W(t) \qquad (12)\end{aligned}$$

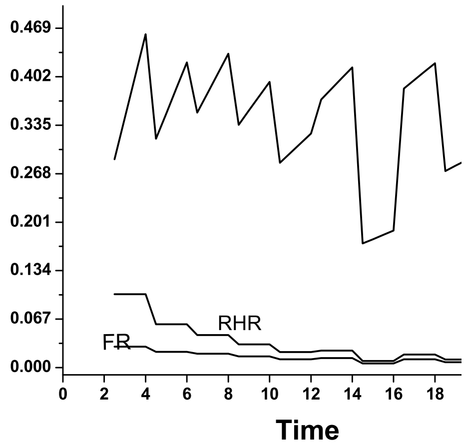

Figure 1: Estimates of $r(t)$, $\mu(t)$, $L(t)$ versus t, plotted for the data in Table 1.

where

$$W(t) = \left\{(\alpha-1)(e^{e^{(\lambda t)^\alpha}} - e) - \{e + (e^{(\lambda t)^\alpha} - 1)e^{e^{(\lambda t)^\alpha}}\}\alpha(\lambda t)^\alpha\right\} \tag{13}$$

Let $p(t) = e^{e^{(\lambda t)^\alpha}} - e$ for $t \geq 0$. Here, $p^{'}(t) = \alpha\lambda(\lambda t)^\alpha e^{e^{(\lambda t)^\alpha}+(\lambda t)^\alpha}$, so that $p(t)$ is a increasing function of t. Thus, $p(t) \geq p(0)$ gives $(e^{e^{(\lambda t)^\alpha}} - e) \geq 0$ for all $t \geq 0$. In (13), we find $W(t) \leq 0$ for $t \geq 0$, if $\alpha < 1$ so that using (12), we conclude $\mu_X(t)$ is a decreasing function of $t \geq 0$. □

2.1 Numerical examples

One can estimate $\bar{F}_X(t)$, $r_X(t)$, $\mu_X(t)$, and $L_X(t)$ with the help of logical estimates, as highlighted in the present section. Let N units be put to test at $t = 0$. Further, let the number of units having survived at ordered times t_j be $N_s(t_j)$. The estimates for $\bar{F}_X(t), r_X(t)$ and $\mu_X(t)$ are respectively given as follows,

$$\hat{\bar{F}}_X(t) = \frac{N_s(t_j)}{N}, \text{ for } t_j < t < t_j + \Delta t_j,$$

$$\hat{r}_X(t) = \frac{\left\{N_s(t_j) - N_s(t_j + \Delta t_j)\right\}}{N_s(t_j)\Delta t_j}, \text{ for } t_j < t < t_j + \Delta t_j$$

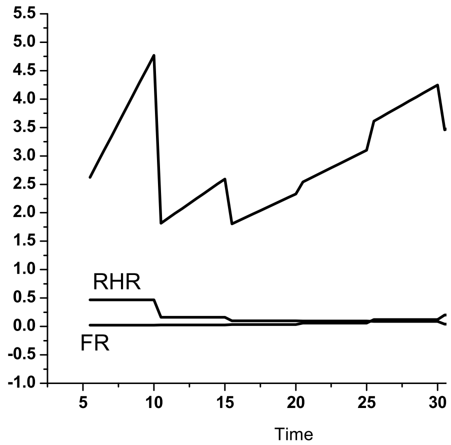

Figure 2: Estimates of $r(t)$, $\mu(t)$, $L(t)$ versus t, plotted for the data in Table 1.

$$\hat{\mu}_X(t) = \frac{\Big\{N_s(t_j) - N_s(t_j + \Delta t_j)\Big\}}{\Big\{N - N_s(t_j)\Big\}\Delta t_j}, \text{ for } t_j < t < t_j + \Delta t_j$$

Thus, a estimate for $L_X(t)$, for $t > 0$, is

$$\hat{L}_X(t) = \frac{-t\Big\{N_s(t_j) - N_s(t_j + \Delta t_j)\Big\}}{N_s(t_j)\Delta t_j \ln \frac{N_s(t_j)}{N}}, \text{ for } t_j < t < t_j + \Delta t_j.$$

A failure data of seventy compressors collected from Ebeling (2004) are observed at 5-month intervals with failures as shown in Table 1. Estimates of $r(t)$, $\mu(t)$, $L(t)$ computed for the failure data in Table 1 are plotted in Figure 1.

The hypothetical data given in Table 2 depict failures in one thousand B-52 bombers (i.e., $N = 1000$) performing various 24-hr missions (cf. Shooman, 1968). $r(t)$, $\mu(t)$, $L(t)$ computed for the failure data in Table 2 are plotted in Figure 2.

3 Concluding Remarks

We look at the system properties of some well known Weibull models, each one of these has wide applications in appropriate scenario. Similar results can be studied for other Weibull models. The comparison of $L(t), h(t), \mu(t)$ is studied for two numerical examples. The Figure 1 and Figure 2 show that the *AI* function considerably differs

Table 1: Failure data of compressors

Time till failure	$N_s(t_j)$	$\frac{N_s(t_j)}{N}$	$N_s(t_j) - N_s(t_j + \Delta t_j)$	$\hat{f}_X(t)$	$\hat{r}_X(t)$	$\hat{\mu}_X(t)$	$\hat{L}_X(t)$
0-5	70	1.00	3	0.0085714	0.0086	-	-
5-10	67	0.957	7	0.02	0.0209	0.4666667	0.47714t
10-15	60	0.857	8	0.022857	0.0267	0.16	0.17321t
15-20	52	0.743	9	0.025714	0.03460	0.1	0.11640t
20-25	43	0.614	13	0.0371429	0.0605	0.0962963	0.12415t
25-30	30	0.429	18	0.051429	0.1200	0.09	0.1416t
30-35	12	0.171	12	0.034286	0.2	0.0413793	0.11341t

Table 2: Failure data of B-52 bombers

Time till failure	$N_s(t_j)$	$\frac{N_s(t_j)}{N}$	$N_s(t_j) - N_s(t_j + \Delta t_j)$	$\hat{f}_X(t_j)$	$\hat{r}_X(t_j)$	$\hat{\mu}_X(t_j)$	$\hat{L}_X(t_j)$
0-2	1000	1	222	0.111	0.111	-	-
2-4	778	0.778	45	0.0225	0.028920308	0.101351	0.1152t
4-6	733	0.733	32	0.016	0.021828104	0.0599251	0.0703t
6-8	701	0.701	27	0.0135	0.019258203	0.045150	0.0542t
8-10	674	0.674	21	0.0105	0.015578635	0.0322086	0.0395t
10-12	653	0.653	15	0.0075	0.011485452	0.0216138	0.0270t
12-14	638	0.638	17	0.0085	0.013322884	0.023481	0.0297t
14-16	621	0.621	7	0.0035	0.005636071	0.009235	0.0118t
16-18	614	0.614	14	0.007	0.011400651	0.018135	0.0234t
18-20	600	0.6	9	0.0045	0.0075	0.01125	0.0147t
20-22	591	0.591	8	0.004	0.00676819	0.00978	0.0129t
22-24	583	0.583	3	0.0015	0.002572899	0.0035971	0.0048t

from failure rate and reversed hazard rate functions for the given data sets. Failure rate function considers only the risk of instantaneous failure whereas aging intensity function, which is the ratio of the instantaneous failure rate to the failure rate average, measures the average risk of failure with time. Sometimes, one may be interested in improving the average aging behavior of a system than that of instantaneous failure rate, which justifies the significant role of AI function in reliability analysis.The present work can be extended for other Weibull distributions to help researchers to conclude about the nature of reversed hazard rate and other aging properties.

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