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Shift point Bayes estimation under Weibull failure model

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The present paper proposes some Bayes estimators for shift point of Weibull failure model under item - failure censoring. The censoring criterion introduced first time in present paper for the shift point estimation. Bayes estimators obtained here for both known as well as unknown shape parameter cases. A simulation study carried out also for analysis of shift point Bayes estimators and their risks.

Keywords: Shift Point Criterion, Bayes Estimation, Item-failure Censored Data.

1 Introduction

The 'time of failure' and 'average life' of a component, measured from some specified time until it fails, is represented by a continuous random variable. Extensively in recent years, one distribution that has been used as a model to deal with such problems for product life is Weibull distribution. The application of the Weibull failure model in life - testing problems and survival analysis has been widely advocated by several authors (Weibull, 1951 ; Berrettoni, 1964). Whittemore and Altschuler (1976) used it as a model in bio-medical applications. It also has been used as model with diverse types of items such as ball bearing (Lieblein and Zelen, 1956), vacuum tube (Kao, 1959), and electrical isolation (Nelson, 1972). Mittnik and Rachev (1993) found that the Weibull distribution might be adequate statistical model for stock returns. Recently, Wahed et al. (2009) consider a new generalization of the Weibull distribution, which incorporates the exponentiated Weibull distribution as a special case and its application in a breast cancer.

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The probability density function of the considered Weibull distribution is given by

$$f(x;v,\theta) = \frac{v}{\theta}\, x^{v-1}\, e^{-\frac{x^v}{\theta}}\;;\; x > 0, v > 0, \theta > 0. \tag{1}$$

Here the parameter v referred as the shape parameter and θ as the scale parameter of Weibull distribution, respectively. When $v = 1$, the Weibull failure model is Exponential distribution and for $v = 2$, it is Rayleigh. Further the values lies in the range $3 \leq v \leq 4$, the shape of the distribution is close to that of Normal distribution and for a large value of v, say $v \geq 10$, is close to that of smallest extreme value distribution. In present article, we study the properties of the Bayes shift point estimator under all four distributions.

Pandey (1983), Pandey et al. (1989), Chandra and Chaudhari (1990) considered the estimation of the Weibull shape parameter in censored data. Singh and Shukla (2000), Tsionas (2002), Prakash and Singh (2008) and others considered the Weibull distribution in different contexts. Recently, Prakash and Singh (2009) present the estimation of the Weibull shape parameter in failure censored sampling criteria.

The aim of the present article is to discuss about Bayes estimation of the shift point for Weibull failure model under item failure censored data. The shift point criterion discussed in Section 2. The Bayes estimators for shift point are obtained in Section 3 when one parameter is known and in Section 5 when both parameters are considered as random variables. A simulation technique carried out in Section 4 and 6 for the illustration of properties of the estimator in terms of posterior risk and Bayes estimate. The sensitivity analysis and conclusion are presented in Section 7 and 8 respectively.

2 The Shift Point

In life testing, fatigue failures and other kinds of destructive test situations, the observations usually occurred in ordered manner such a way that weakest items failed first and then second one and so on. Let us suppose that n items are put to test under the model without replacement and the test terminates as soon as first r^{th} $(r \leq n)$ item fails. This censoring scheme is known as item - failure censoring scheme.

In order to obtain the information on their endurance, manufactured items such as mechanical or electronic components are often put to life tests and life times observed periodically. Physical systems manufacturing the items are often subject to random fluctuations. It may happen that at some point of time, there is a change in the parameter. The objective of study is to find out when and where this change has started occurring. This estimation process is called as the shift point inference problem. The Bayesian model plays an important role in the study of such estimation problem and has been extensively studied by Broemeling and Tsurumi (1987), Jani and Pandya (1999), Ebrahimi and Ghosh (2001). Recently, Pandya and Jadav (2010) presents Bayesian estimation of shift point in mixture of left truncated exponential and degenerate distribution.

We are introducing first time the censoring criteria under shift point estimation. For this let us first assume that a sequence of ordered random sample of size n such as $x_{(1)}, x_{(2)}, ..., x_{(r-1)}, x_{(r)}, x_{(r+1)}, ..., x_{(n)}$ from the model (1) with parameters θ_1 and v. All n items are put to test and the test terminates as soon as first r^{th} items fails.

We have a sequence of $r\,(\leq n)$ ordered random sample $x_{(1)}, x_{(2)}, ..., x_{(m-1)}, x_{(m)}\ , x_{(m+1)}, ..., x_{(r)}$ from assumed sample of size n with survival function $\Psi_1\,(t)$ at any mission time $t(>0)$, but later it is found that there is a change in the system at some point of time $m(\leq r)$ and it is reflected in the sequence after the item $x_{(m)}$ by the change in survival function $\Psi_2\,(t)$.

Thus, first m random observations $x_{(1)}, x_{(2)}, ..., x_{(m)}$ follow the model (1) with probability density function

$$f\left(x_{(i)}; v, \theta_1\right) = \frac{v}{\theta_1}\, x_{(i)}^{v-1}\, e^{-\frac{x_{(i)}^{v}}{\theta_1}}\ ;\ x_{(i)} > 0, v > 0, \theta_1 > 0, \\ i = 1, 2, ..., m\,(m \leq r, r \leq n) \tag{2}$$

with survival function

$$\Psi_1\left(x_{(i)}\right) = exp\left(-\frac{x_{(i)}^{v}}{\theta_1}\right). \tag{3}$$

First remaining $(r-m)$ components $x_{(m+1)}, x_{(m+2)}, ..., x_{(r)}$ from a sample of size r follow the model (1) with the probability density function

$$f\left(x_{(i)}; v, \theta_2\right) = \frac{v}{\theta_2}\, x_{(i)}^{v-1}\, e^{-\frac{x_{(i)}^{v}}{\theta_2}}\ ;\ x_{(i)} > 0, v > 0, \theta_2 > 0, \\ i = m+1, m+2, ..., r\,(m \leq r, r \leq n) \tag{4}$$

with survival function

$$\Psi_2\left(x_{(i)}\right) = exp\left(-\frac{x_{(i)}^{v}}{\theta_2}\right). \tag{5}$$

The last remaining group of the random samples $x_{(r+1)}, x_{(r+2)}, ..., x_{(n)}$ of size $(n-r)$ follows the Weibull model with parameters v and θ_2 and having the probability density function

$$f\left(x_{(i)}; v, \theta_2\right) = \frac{v}{\theta_2}\, x_{(i)}^{v-1}\, e^{-\frac{x_{(i)}^{v}}{\theta_2}}\ ;\ x_{(i)} > 0, v > 0, \theta_2 > 0, \\ i = r+1, r+2, ..., n\,(n \geq r)\,. \tag{6}$$

The likelihood function for the shift point under item - failure censoring criterion is defined as

$$L\left(\theta_1, \theta_2, m|x_{(1)}, x_{(2)}, ..., x_{(r)}\right) = \left(\prod_{i=1}^{m} f\left(x_{(i)}\,; v, \theta_1\right)\right).$$

$$\left(\prod_{i=m+1}^{r} f\left(x_{(i)}\,; v, \theta_2\right)\right).\left(exp\left(-\frac{x_{(r)}^{v}}{\theta_2}\right)^{n-r}\right). \tag{7}$$

Solving (7) we have

$$L\left(\theta_1, \theta_2, m|x_{(1)}, x_{(2)}, ..., x_{(r)}\right) = \frac{v^r}{\theta_1^m \theta_2^{r-m}}\left(\prod_{i=1}^{r} x_{(i)}^{v-1}\right)\left(exp\left(-\frac{\delta_1}{\theta_1} - \frac{\delta_2}{\theta_2}\right)\right); \tag{8}$$

where $\delta_1 = \sum_{i=1}^{m} x_{(i)}^{v}$ and $\delta_2 = \sum_{i=m+1}^{r} x_{(i)}^{v} - (n-r)\, x_{(i)}^{v}$.

Substituting $\theta_1 = \theta = \theta_2$ in (8) we have

$$L\left(\theta|x_{(1)}, x_{(2)}, ..., x_{(r)}\right) = \left(\frac{v}{\theta}\right)^r \left(\prod_{i=1}^{r} x_{(i)}^{v-1}\right)\left(exp\left(-\frac{\delta_3}{\theta}\right)\right); \delta_3 = \delta_1 + \delta_2. \tag{9}$$

Equation (9) shows the likelihood function under the item - failure censoring criterion without shift point.

Similarly, the likelihood function without shift point under the complete sample case is obtain by substituting $\theta_1 = \theta = \theta_2$ and $r = n$ in (8) i.e.

$$L\left(\theta|x_{(1)}, x_{(2)}, ..., x_{(n)}\right) = \left(\frac{v}{\theta}\right)^n \left(\prod_{i=1}^{n} x_{(i)}^{v-1}\right)\left(exp\left(-\sum_{i=1}^{n}\frac{x_{(i)}^{v}}{\theta}\right)\right). \tag{10}$$

3 Bayes Estimator for Shift Point (Shape Parameter Known)

We believe, as stated in Arnold and Press (1983) that from a Bayesian viewpoint, there is clearly no way in which one can say that one prior is better than other. It is more frequently the case that, we select to restrict attention to a given flexible family of priors, and we choose one from that family, which seems to match best with our personal beliefs. One of the best choices of selecting the prior distribution is the conjugate prior. Thus in the present case we considered the inverted Gamma distribution as the natural family of conjugate prior for the parameter θ and defined as

$$g(\theta) \propto \theta^{-(\alpha+1)}\, exp\left(-\frac{\beta}{\theta}\right)\;;\ \alpha > 0, \beta > 0, \theta > 0. \tag{11}$$

Under the shift point criterion, the prior information regarding the parameter θ is re - define as

$$g_j(\theta_j) \propto \theta_j^{-(\alpha_j+1)}\, exp\left(-\frac{\beta_j}{\theta_j}\right)\ ;\ \alpha_j > 0, \beta_j > 0, \theta_j > 0,\ j = 1, 2. \tag{12}$$

The prior distribution for the shift point m is considered as discrete uniform over the set $(1, 2, ..., r-1)$ and defined as

$$g_3(m) = \frac{1}{r-1}; r > 0. \tag{13}$$

The joint prior distribution is thus defined as

$$h(\theta_1, \theta_2, m) \propto g_1(\theta_1)\ .\ g_2(\theta_2)\ .\ g_3(m).$$

Hence, the joint posterior density is defined as

$$Z(\theta_1, \theta_2, m) = \frac{L\left(\theta_1, \theta_2, m|x_{(1)}, x_{(2)}, ..., x_{(r)}\right) . h(\theta_1, \theta_2, m)}{\sum_m \int_{\theta_1}\int_{\theta_2} L\left(\theta_1, \theta_2, m|x_{(1)}, x_{(2)}, ..., x_{(r)}\right) . h(\theta_1, \theta_2, m)\, d\theta_2 d\theta_1}.$$

After simplification, the joint posterior density is

$$Z(\theta_1, \theta_2, m) = \Phi\, \theta_1^{-(m+\alpha_1+1)}\, \theta_2^{-(r-m+\alpha_2+1)}\, exp\left(-\frac{\omega_1}{\theta_1} - \frac{\omega_2}{\theta_2}\right)\ ; \tag{14}$$

where $\Phi = \left(\sum_m \Delta\right)^{-1}$, $\Delta = \left(\frac{\Gamma(m+\alpha_1)\,\Gamma(r-m+\alpha_2)}{\omega_1^{m+\alpha_1}\,\omega_2^{r-m+\alpha_2}}\right)$ and $\omega_j = \beta_j + \delta_j\ ; j = 1, 2.$

Now, the marginal posterior density for shift point m is obtain as

$$Z^*(m) = \Phi \int_{\theta_1}\int_{\theta_2} exp\left(-\frac{\omega_1}{\theta_1} - \frac{\omega_2}{\theta_2}\right) \theta_1^{-(m+\alpha_1+1)}\, \theta_2^{-(r-m+\alpha_2+1)}\, d\theta_2\, d\theta_1$$

$$\Rightarrow Z^*(m) = \Phi\, \Delta. \tag{15}$$

The Bayes estimator for shift point m under the squared error loss function (SELF) is simply the posterior mean and obtained as

$$\hat{m}_S = E_p(m) = \Phi\left(\sum_m m\Delta\right). \tag{16}$$

Here, the suffix p indicates the expectation taken under the posterior density. The posterior risk for the Bayes estimator $\hat{m}_S$ is obtained as

$$R_{(S)}(\hat{m}_S) = \int_0^\infty e^{-z}\, z^{n-1}\left(w_{(S)}\right)\, dz,$$

where $w_{(s)} = (\hat{m}_S - m)^2 |_{(z)}$ be the function of z.

The choice of the loss function may be crucial. It has always been recognized that the most commonly used loss function, squared error loss function (SELF) is in appropriate in many situations. If the SELF is taken as a measure of inaccuracy then the resulting risk is often too sensitive to the assumptions about the behavior of tail of the probability distribution. In addition, in some estimation problems overestimation is more serious than the underestimation, or vice - versa. To deal with such cases, a useful and flexible class of asymmetric loss function (LINEX loss function (LLF)) is given as

$$L(\partial) = e^{a\partial} - a\partial - 1\,;\; \partial = \hat{\theta} - \theta. \tag{17}$$

Here $'a'$ is the shape parameter of LLF and $\hat{\theta}$ is any estimate of the unknown parameter θ. The negative (positive) value of $'a'$, gives more weight to overestimation (underestimation) and its magnitude reflect the degree of asymmetry. It is also seen that, for $a = 1$, the function is quite asymmetric with overestimation being costly than underestimation (See Parsian and Kirmani (2002), Singh et al. (2007)). For small values of $|a|$, the LLF is almost symmetric and is not far from the SELF.

The Bayes estimator for the shift point m under LLF defined above is obtain as

$$\hat{m}_L = -\frac{1}{a} ln\, E_p\left\{e^{-am}\right\} = -\frac{1}{a} ln\, \left\{\Phi \sum_m \Delta e^{-am}\right\}. \tag{18}$$

Similarly, the posterior risk for the Bayes estimator $\hat{m}_L$ under LLF is

$$R_{(L)}\left(\hat{m}_L\right) = \int_0^\infty e^{-z}\, z^{n-1} \left(e^{w_{(L)}} - w_{(L)}\right) dz - 1;$$

where $w_{(L)} = a\left(\hat{m}_L - m\right)|_{(z)}$ be the function of z.

4 Numerical Analysis (Shape Parameter Known)

To assess and study the properties of the Bayes estimators for shift point m when the shape parameter v is known, a simulation study has been carried out. The random samples are generated as follows:

1. Generate θ_1 and θ_2 through prior density $g_1(\theta_1)$ and $g_2(\theta_2)$ for the given values of prior parameters α_j and β_j as $(\alpha_j, \beta_j) = (03, 02), (06, 10), (11, 30)$, $j = 1, 2$. The value of α_j and β_j are chosen so as to keep the prior variance unity.

2. Using θ_1 and θ_2 obtained in (1), and the considered values of shape parameter $v = 1.00, 2.00, 3.50, 12.00$; generate the $10,000$ random samples of size $n = 15$ from the model (2) and (4).

3. Here the value of $v = 1.00$ and 2.00 should meet the criterion for the Exponential and Rayleigh distribution respectively. Similarly others two values make the shape of the distribution close to that of Normal and smallest extreme value distribution respectively.

4. For the selected set of censored sample size $r = 04, 06, 08, 10$; the values of the Bayes estimate and posterior risk for the shift point under the SELF have been obtained and presented in the Table 01 - 02.

5. It is observed here that when censored sample size r increases the magnitude of the Bayes estimator increases but the increment in magnitude is nominal (robust).

6. A decreasing trend has been seen when (α_j, β_j) increases.

7. It is also noted that when the values of the shape parameter v increases the magnitude of the Bayes estimator also increases, however for large value of shape parameter v (say 12) i.e., for the smallest extreme value distribution the magnitude of the Bayes estimator decreases.

8. With considered set of values and $a = 0.25, 0.50, 1.00, 2.00$; the magnitude of the Bayes estimator under LLF and posterior risk have been obtained and presented in the Table 03 - 04, only for $a = 0.25, 1.00$.

9. Similar properties as discussed above have been seen for the Bayes estimate of the shift point $\hat{m}_L$. Further, an increasing trend in the magnitude of the estimate also has been seen when $'a'$ increases (except for large v) but the increment in magnitude is robust.

10. It observed from tables that the magnitudes of posterior risk are smaller and nominal. Other properties are seen to be similar as discussed above.

Remark:

In the case when the censored sample size $r = 15$; the censoring criterion is reduces to the complete sample size criterion and hence the result are valid for complete sample case.

5 Bayes Estimator of Shift Point (Shape Parameter Unknown)

When both of the parameters θ and v of the considered model (1) are unknown, there do not exist any joint conjugate prior distribution. One of the good choices of the joint prior distribution when both parameters are unknown is given by

$$g(\theta, v) = g(\theta|v) \ . f(v). \tag{19}$$

The prior distributions $g\left(\theta|v\right)$ and $f(v)$ are defined as

$$g\left(\theta|v\right)=\frac{v^{\alpha}}{\Gamma(\alpha)}\,\theta^{-(\alpha+1)}\,exp\left(-\frac{v}{\theta}\right)\ ;\ \alpha>0,v>0,\theta>0$$

and

$$f(v)=\frac{b^{c}}{\Gamma(c)}\,\theta^{-(c+1)}\,exp\left(-\frac{b}{v}\right)\ ;\ b>0,c>0,v>0.$$

When both parameters are considered to be unknown, the likelihood function is given as

$$L\left(\theta_1,\theta_2,v,m|x_{(1)},x_{(2)},...,x_{(r)}\right)=\frac{v^{r}}{\theta_1^{m}\,\theta_2^{r-m}}\left(\prod_{i=1}^{r}x_{(i)}^{v-1}\right)\left(exp\left(-\frac{\delta_1}{\theta_1}-\frac{\delta_2}{\theta_2}\right)\right). \tag{20}$$

The joint prior distribution is now defined as

$$h_1\left(\theta_1,\theta_2,v,m\right)\propto g_1\left(\theta_1|v\right)\ .\ g_2\left(\theta_2|v\right)\ .\ f(v)\ .\ g_3\left(m\right);$$

where $g_j\left(\theta_j|v\right)=\frac{v^{\alpha_j}}{\Gamma(\alpha_j)}\,\theta_j^{-(\alpha_j+1)}\,exp\left(-\frac{v}{\theta_j}\right)\ ;\ \alpha_j>0,v>0,\theta_j>0,\ j=1,2.$

Thus, the joint posterior density is obtained as

$$Z_1\left(\theta_1,\theta_2,v,m\right)=\frac{1}{K_1}\,L\left(\theta_1,\theta_2,v,m|x_{(1)},x_{(2)},...,x_{(r)}\right)\ .\ h_1\left(\theta_1,\theta_2,v,m\right)$$

where $K_1=\sum_m\int_v\int_{\theta_1}\int_{\theta_2}\,L\left(\theta_1,\theta_2,v,m|x_{(1)},x_{(2)},...,x_{(r)}\right).h_1\left(\theta_1,\theta_2,v,m\right).d\theta_2.d\theta_1.dv$

After simplification the joint posterior density is

$$Z_1\left(\theta_1,\theta_2,v,m\right)=\bar{\Phi}\,\theta_1^{-(m+\alpha_1+1)}\,\theta_2^{-(r-m+\alpha_2+1)}\,\xi(v)\,exp\left(-\frac{\omega_3}{\theta_1}-\frac{\omega_4}{\theta_2}\right); \tag{21}$$

where $\bar{\Phi}=\left(\sum_m\bar{\bar{\Delta}}\right)^{-1}$, $\bar{\bar{\Delta}}=\int_v\xi(v)\bar{\Delta}dv$, $\bar{\Delta}=\left(\frac{\Gamma(m+\alpha_1)\,\Gamma(r-m+\alpha_2)}{\omega_3^{m+\alpha_1}\,\omega_4^{r-m+\alpha_2}}\right)$, $\xi(v)=v^{r+\alpha_1+\alpha_2-c-1}$ $\prod_{i=1}^{r}x_{(i)}^{v-1}\,exp\left(-\frac{b}{v}\right)$ and $\omega_{j+2}=v+\delta_j;j=1,2.$

The marginal posterior density for shift point m in present case is obtain as

$$Z_1^{*}(m)=\int_{\theta_1}\int_{\theta_2}\int_{v}Z_1\left(\theta_1,\theta_2,v,m\right)\ dv.\ d\theta_2.\ d\theta_1$$

$$\Rightarrow Z_1^{*}(m)=\bar{\phi}\bar{\bar{\Delta}}. \tag{22}$$

The Bayes estimator for the shift point estimator m under the SELF is

$$\hat{m}_{S1}=\bar{\phi}\sum_{m}\left(m\,\bar{\bar{\Delta}}\right). \tag{23}$$

Similarly, the Bayes estimator for the shift point under the LLF is given by

$$\hat{m}_{L1} = -\frac{1}{a} ln \left\{ \bar{\phi} \sum_{m} \left(e^{-am} \, \bar{\bar{\Delta}} \right) \right\}. \tag{24}$$

The posterior risk for Bayes estimators $\hat{m}_{S1}$ and $\hat{m}_{L1}$ are obtained similarly as in known shape parameter case.

6 Numerical Analysis (Shape Parameter Unknown)

When both parameters are considered as the random variable, a simulation study also has been carried out to study the properties of the Bayes estimators for shift point as follows:

1. Generate the values of the shape parameter v through prior density $f(v)$ for the given set of values $(c, b) = (03, 02), (06, 10), (11, 30)$. The value of c and b are chosen so as to keep the prior variance unity.

2. Using the generated values of v in (1), we generate the values of θ_1 and θ_2 for the previous selected values of prior parameter.

3. Using above generated values of θ_1, θ_2 and v obtained in steps (1) & (2), generate the $10,000$ random samples of size $n = 15$ form the considered model.

4. The values of Bayes estimate $\hat{m}_{S1}$ for shift point under SELF and Posterior risk has been obtain for censored sample size $r = 04, 06, 08, 10$, and presented in Tables 05 - 06.

5. All the properties are seen similar as compared to $\hat{m}_S$ (known shape parameter case).

6. Using similar set of parametric values as discussed earlier, the magnitude of Bayes estimator $\hat{m}_{L1}$ under LLF and Posterior risk have been obtained and presented in Tables 07 - 08, only for $a = 0.25, 1.00$.

7. An increasing trend in the magnitude of the estimate also has been seen when $'a'$ increases but the increment in magnitude is least (robust). Others properties are similar as in case of known shape parameter.

8. Both the estimators are robust and the magnitudes of posterior risk are least. Other properties are seen to be similar as discussed above

7 Sensitivity of Bayes Estimates

Following Calabria and Pulcini (1996), we study the sensitivity of the Bayes estimator with respect to change in prior of parameters. The prior mean and prior variance

$\left(\mu_j, \sigma_j^2\ ;\ j = 1, 2\right)$ have been used as the prior information in computing the hyper - parameters of the prior distribution. The sensitivity analysis is based on assumption that the prior information to be correct if the true value of the parameters $\theta_1\,(\theta_2)$ is close to prior mean $\mu_1\,(\mu_2)$ and is assumed to be wrong if the parameters $\theta_1\,(\theta_2)$ is far from the prior mean $\mu_1\,(\mu_2)$. For this, we have computed the posterior mean for the selected set of parameters as discussed in section 4 and presented in Table 09. It is seen from the table that the posterior mean appears to be robust with respect to the correct choice as well as wrong choice of the prior density of $\theta_1\,(\theta_2)$.

8 Conclusion

Two parameter Weibull distribution is consider here as the underlying model for the study. The study of shift point estimation for the Weibull model is performing under the Bayesian approach. The censoring criteria inside the shift point estimation have been proposed first time in present article. Both known and unknown case of shape parameter is considered here for the study of Bayes estimation of the shift point under the symmetric and asymmetric loss function.

A simulation study has been carried out for the study of the properties of the Bayes estimation. Based on the findings the magnitude of the estimator is robust. It is also observed that when the value of the shape parameter of LLF is small the difference between the magnitudes of Bayes estimates under SELF and LLF is robust.

Table 1: Bayes Estimate of $\hat{m}_S$ (Shape Parameter Known)

$n = 15$		$r \downarrow$				
v	$(\beta_j, \alpha_j)\,; j = 1, 2 \downarrow$	4	6	8	10	15
1.00	02, 03	4.6848	4.6990	4.7084	4.7178	4.7271
1.00	10, 06	4.6449	4.6588	4.6681	4.6774	4.6867
1.00	30, 11	4.1847	4.1973	4.2057	4.2140	4.2223
2.00	02, 03	4.7112	4.7252	4.7347	4.7442	4.7537
2.00	10, 06	4.6910	4.7053	4.7146	4.7239	4.7331
2.00	30, 11	4.6908	4.7049	4.7143	4.7237	4.7330
3.50	02, 03	4.8053	4.8198	4.8294	4.8390	4.8487
3.50	10, 06	4.7849	4.7993	4.8088	4.8183	4.8278
3.50	30, 11	4.7002	4.7143	4.7237	4.7330	4.7424
12.00	02, 03	4.5584	4.5720	4.5812	4.5904	4.5996
12.00	10, 06	4.5389	4.5527	4.5617	4.5707	4.5797
12.00	30, 11	4.4587	4.4719	4.4808	4.4897	4.4986

Table 2: Posterior Risk for $\hat{m}_S$ (Shape Parameter Known)

$n = 15$		$r \downarrow$				
v	$(\beta_j, \alpha_j)\,; j = 1, 2 \downarrow$	4	6	8	10	15
1.00	02, 03	1.6709	1.6758	1.6792	1.6826	1.6860
1.00	10, 06	1.6133	1.6182	1.6214	1.6246	1.6278
1.00	30, 11	1.5352	1.5398	1.5428	1.5459	1.5490
2.00	02, 03	1.7614	1.7667	1.7702	1.7737	1.7773
2.00	10, 06	1.7007	1.7058	1.7092	1.7126	1.7160
2.00	30, 11	1.6184	1.6232	1.6265	1.6297	1.6329
3.50	02, 03	1.7640	1.7694	1.7729	1.7765	1.7800
3.50	10, 06	1.7142	1.7195	1.7229	1.7263	1.7296
3.50	30, 11	1.6620	1.6670	1.6704	1.6737	1.6770
12.00	02, 03	1.7740	1.7792	1.7828	1.7864	1.7900
12.00	10, 06	1.6974	1.7025	1.7059	1.7093	1.7127
12.00	30, 11	1.4827	1.4872	1.4902	1.4931	1.4960

Table 3: Bayes Estimate of $\hat{m}_L$ (Shape Parameter Known)

$n = 15, a = 0.25$		$r \downarrow$				
v	$(\beta_j, \alpha_j)\,; j = 1, 2 \downarrow$	4	6	8	10	15
1.00	02, 03	4.3086	4.3215	4.3301	4.3387	4.3475
1.00	10, 06	4.2718	4.2846	4.2931	4.3016	4.3102
1.00	30, 11	3.8487	3.8602	3.8678	3.8755	3.8832
2.00	02, 03	4.3800	4.3931	4.4018	4.4106	4.4195
2.00	10, 06	4.3612	4.3744	4.3831	4.3918	4.4004
2.00	30, 11	4.3610	4.3740	4.3828	4.3916	4.4003
3.50	02, 03	4.5302	4.5437	4.5528	4.5619	4.5710
3.50	10, 06	4.5108	4.5245	4.5334	4.5423	4.5512
3.50	30, 11	4.4311	4.4443	4.4531	4.4619	4.4707
12.00	02, 03	4.2471	4.2597	4.2682	4.2769	4.2855
12.00	10, 06	4.2290	4.2418	4.2501	4.2584	4.2670
12.00	30, 11	4.1542	4.1665	4.1748	4.1831	4.1914
$n = 15, a = 1.00$						
1.00	02, 03	4.9367	4.9514	4.9613	4.9712	4.9812
1.00	10, 06	4.8945	4.9092	4.9188	4.9287	4.9385
1.00	30, 11	4.4096	4.4229	4.4316	4.4404	4.4493
2.00	02, 03	5.0118	5.0267	5.0367	5.0468	5.0570
2.00	10, 06	4.9903	5.0054	5.0153	5.0252	5.0351
2.00	30, 11	4.9901	5.0050	5.0151	5.0249	5.0350
3.50	02, 03	5.7031	5.7200	5.7316	5.7430	5.7544
3.50	10, 06	5.6787	5.6959	5.7072	5.7184	5.7296
3.50	30, 11	5.5783	5.5949	5.6059	5.6172	5.6283
12.00	02, 03	4.0208	4.0328	4.0409	4.0491	4.0572
12.00	10, 06	4.0037	4.0158	4.0237	4.0316	4.0396
12.00	30, 11	3.9328	3.9445	3.9524	3.9603	3.9681

Table 4: Posterior Risk for $\hat{m}_L$ (Shape Parameter Known)

$n = 15, a = 0.25$		$r \downarrow$				
v	$(\beta_j, \alpha_j)\,; j = 1, 2 \downarrow$	4	6	8	10	15
1.00	02, 03	1.3467	1.3507	1.3534	1.3562	1.3589
1.00	10, 06	1.3394	1.3434	1.3460	1.3487	1.3514
1.00	30, 11	1.3114	1.3152	1.3179	1.3205	1.3231
2.00	02, 03	1.4365	1.4408	1.4437	1.4466	1.4495
2.00	10, 06	1.4286	1.4330	1.4358	1.4386	1.4414
2.00	30, 11	1.3988	1.4029	1.4057	1.4085	1.4113
3.50	02, 03	1.5984	1.6032	1.6064	1.6096	1.6129
3.50	10, 06	1.5682	1.5729	1.5760	1.5791	1.5823
3.50	30, 11	1.3729	1.3770	1.3797	1.3824	1.3852
12.00	02, 03	1.6249	1.6298	1.6330	1.6363	1.6396
12.00	10, 06	1.6010	1.6058	1.6090	1.6122	1.6154
12.00	30, 11	1.5556	1.5602	1.5634	1.5665	1.5696
$n = 15, a = 1.00$						
1.00	02, 03	1.3573	1.3614	1.3641	1.3669	1.3696
1.00	10, 06	1.3446	1.3486	1.3512	1.3540	1.3567
1.00	30, 11	1.3137	1.3176	1.3203	1.3229	1.3255
2.00	02, 03	1.4478	1.4522	1.4551	1.4580	1.4609
2.00	10, 06	1.4342	1.4386	1.4414	1.4442	1.4470
2.00	30, 11	1.4013	1.4054	1.4082	1.4110	1.4138
3.50	02, 03	1.6110	1.6159	1.6191	1.6223	1.6256
3.50	10, 06	1.5743	1.5790	1.5821	1.5853	1.5885
3.50	30, 11	1.3754	1.3795	1.3822	1.3849	1.3877
12.00	02, 03	1.6377	1.6427	1.6459	1.6492	1.6525
12.00	10, 06	1.6072	1.6121	1.6153	1.6185	1.6217
12.00	30, 11	1.5584	1.5630	1.5662	1.5693	1.5724

Table 5: Bayes Estimate of $\hat{m}_{S1}$ (Shape Parameter Unknown)

$n = 15$		$r \downarrow$				
$\alpha_1 = \alpha_2$	$(b, c) \downarrow$	4	6	8	10	15
1.00	02, 03	4.3800	4.3932	4.4019	4.4108	4.4195
1.00	10, 06	4.3427	4.3556	4.3643	4.3730	4.3817
1.00	30, 11	3.9125	3.9242	3.9320	3.9398	3.9476
2.00	02, 03	4.4502	4.4635	4.4725	4.4814	4.4904
2.00	10, 06	4.4312	4.4447	4.4534	4.4622	4.4709
2.00	30, 11	4.4310	4.4443	4.4531	4.4620	4.4708
3.50	02, 03	4.6300	4.6438	4.6531	4.6625	4.6717
3.50	10, 06	4.6102	4.6241	4.6333	4.6424	4.6516
3.50	30, 11	4.5287	4.5422	4.5511	4.5602	4.5693
12.00	02, 03	4.2618	4.2745	4.2830	4.2917	4.3002
12.00	10, 06	4.2436	4.2564	4.2648	4.2732	4.2817
12.00	30, 11	4.1685	4.1810	4.1892	4.1976	4.2060

Table 6: Posterior Risk for $\hat{m}_{S1}$ (Shape Parameter Unknown)

$n = 15$		$r \downarrow$				
$\alpha_1 = \alpha_2$	$(b, c) \downarrow$	4	6	8	10	15
1.00	02, 03	1.0448	1.0480	1.0501	1.0522	1.0543
1.00	10, 06	0.9491	0.9519	0.9538	0.9557	0.9576
1.00	30, 11	0.8159	0.8184	0.8200	0.8216	0.8233
2.00	02, 03	1.1061	1.1094	1.1116	1.1138	1.1161
2.00	10, 06	1.0570	1.0603	1.0623	1.0644	1.0665
2.00	30, 11	0.9033	0.9060	0.9078	0.9096	0.9114
3.50	02, 03	1.1971	1.2006	1.2030	1.2055	1.2078
3.50	10, 06	1.0997	1.1031	1.1053	1.1074	1.1096
3.50	30, 11	0.9897	0.9927	0.9946	0.9966	0.9986
12.00	02, 03	1.1871	1.1906	1.1930	1.1954	1.1978
12.00	10, 06	1.1749	1.1784	1.1807	1.1831	1.1854
12.00	30, 11	0.8915	0.8941	0.8959	0.8977	0.8995

Table 7: Bayes Estimate of $\hat{m}_{L1}$ (Shape Parameter Unknown)

$n = 15, a = 0.25$		$r \downarrow$				
$\alpha_1 = \alpha_2$	$(b, c) \downarrow$	4	6	8	10	15
1.00	02, 03	4.1015	4.1137	4.1220	4.1303	4.1385
1.00	10, 06	4.0665	4.0787	4.0867	4.0948	4.1030
1.00	30, 11	3.6636	3.6746	3.6819	3.6892	3.6966
2.00	02, 03	4.1164	4.1286	4.1368	4.1451	4.1534
2.00	10, 06	4.0987	4.1111	4.1193	4.1274	4.1355
2.00	30, 11	4.0985	4.1108	4.1190	4.1272	4.1354
3.50	02, 03	4.5857	4.5995	4.6087	4.6179	4.6272
3.50	10, 06	4.5663	4.5800	4.5891	4.5981	4.6072
3.50	30, 11	4.4855	4.4988	4.5077	4.5167	4.5256
12.00	02, 03	3.3406	3.3505	3.3573	3.3640	3.3708
12.00	10, 06	3.3264	3.3364	3.3429	3.3496	3.3563
12.00	30, 11	3.2675	3.2772	3.2838	3.2903	3.2968
$n = 15, a = 1.00$						
1.00	02, 03	4.5248	4.5382	4.5474	4.5565	4.5656
1.00	10, 06	4.4862	4.4996	4.5085	4.5174	4.5264
1.00	30, 11	4.0417	4.0538	4.0619	4.0699	4.0781
2.00	02, 03	4.5412	4.5547	4.5637	4.5729	4.5821
2.00	10, 06	4.5217	4.5354	4.5444	4.5534	4.5623
2.00	30, 11	4.5215	4.5350	4.5441	4.5531	4.5622
3.50	02, 03	5.1171	5.1324	5.1427	5.1529	5.1632
3.50	10, 06	5.0952	5.1106	5.1208	5.1308	5.1410
3.50	30, 11	5.0051	5.0200	5.0299	5.0400	5.0500
12.00	02, 03	3.6787	3.6896	3.6970	3.7045	3.7120
12.00	10, 06	3.6630	3.6740	3.6812	3.6885	3.6959
12.00	30, 11	3.5981	3.6088	3.6161	3.6233	3.6305

Table 8: Posterior Risk for $\hat{m}_{L1}$ (Shape Parameter Unknown)

$n = 15, a = 0.25$		$r\downarrow$				
$\alpha_1 = \alpha_2$	$(b, c)\downarrow$	4	6	8	10	15
1.00	02, 03	1.2703	1.2741	1.2766	1.2792	1.2817
1.00	10, 06	1.2594	1.2632	1.2657	1.2682	1.2707
1.00	30, 11	1.1347	1.1381	1.1403	1.1425	1.1449
2.00	02, 03	1.2749	1.2787	1.2812	1.2838	1.2864
2.00	10, 06	1.2694	1.2732	1.2758	1.2783	1.2808
2.00	30, 11	1.2694	1.2732	1.2757	1.2782	1.2808
3.50	02, 03	1.4365	1.4409	1.4437	1.4466	1.4495
3.50	10, 06	1.4305	1.4348	1.4376	1.4404	1.4433
3.50	30, 11	1.4051	1.4093	1.4121	1.4149	1.4177
12.00	02, 03	1.0328	1.0358	1.0379	1.0400	1.0421
12.00	10, 06	1.0283	1.0314	1.0335	1.0355	1.0376
12.00	30, 11	1.0101	1.0132	1.0152	1.0172	1.0192
$n = 15, a = 1.00$						
1.00	02, 03	1.3526	1.3566	1.3593	1.3621	1.3648
1.00	10, 06	1.3410	1.3450	1.3477	1.3504	1.3531
1.00	30, 11	1.2082	1.2118	1.2142	1.2166	1.2190
2.00	02, 03	1.3575	1.3615	1.3642	1.3670	1.3697
2.00	10, 06	1.3516	1.3557	1.3584	1.3611	1.3638
2.00	30, 11	1.3516	1.3557	1.3584	1.3611	1.3638
3.50	02, 03	1.5123	1.5168	1.5198	1.5229	1.5259
3.50	10, 06	1.5058	1.5104	1.5133	1.5163	1.5193
3.50	30, 11	1.4792	1.4836	1.4865	1.4895	1.4924
12.00	02, 03	1.1017	1.1049	1.1072	1.1094	1.1116
12.00	10, 06	1.0970	1.1003	1.1024	1.1046	1.1068
12.00	30, 11	1.0775	1.0808	1.0829	1.0851	1.0872

Table 9: Estimate of Posterior Mean

$n = 15$		Posterior Mean			
μ_1	μ_2	4	6	8	10
1.00	1.00	6.9851	7.9328	8.2158	9.3305
2.00	2.00	6.9855	7.9333	8.2163	9.3311
3.00	3.00	6.9862	7.9341	8.2169	9.3317

References

Arnold, B. C. and Press, S. J. (1983). Bayesian inference for pareto populations. *Journal of Econometrics*, 21:287–306.

Berrettoni, J. A. (1964). Practical applications of the weibull distribution. *Industrial Quality Control*, 21:71–79.

Broemeling, L. D. and Tsurumi, H. (1987). *Econometrics and structural change.* Marcel Dekker, New York.

Calabria, R. and Pulcini, G. (1996). Point estimation under asymmetric loss functions for left - truncated exponential samples. *Communication in Statistics - Theory and Methods*, 25(3):585–600.

Chandra, N. K. and Chaudhari, A. (1990). On testing the weibull shape parameter. *Communications in Statistics - Computation and Simulation*, 19(2):637–648.

Ebrahimi, N. and Ghosh, S. K. (2001). Bayesian and frequentist methods in change - point problems. *Handbook of statistics: Advance in Reliability, Eds. N. Balakrishna & C. R. Rao*, 20:777–787.

Jani, P. N. and Pandya, M. (1999). Bayes estimation of shift point in left truncated exponential sequence. *Communications in Statistics - Theory and Methods*, 28(11):2623–2639.

Kao, J. H. K. (1959). A graphical estimation of mixed weibull parameters in life testing electron tubes. *Technometrics*, 4:309–407.

Lieblein, J. and Zelen, M. (1956). Statistical investigation of the fatigue life of deep groove ball bearings. *Journal of Research: National Bureau of Standards*, 57:273–315.

Mittnik, S. and Rachev, S. T. (1993). Modeling asset returns with alternative stable distribution. *Economic Reviews*, 12:261–330.

Nelson, W. B. (1972). Graphical analysis of accelerated life test data with the inverse power law model. *IEEE Transaction on Reliability*, 21(R):2–11.

Pandey, B. N., Malik, H. J., and Srivastava, R. (1989). Shrinkage testimators for the shape parameter of weibull distribution under type - ii censoring. *Communications in Statistics - Theory and Methods*, 18(6):1175–1199.

Pandey, M. (1983). Shrunken estimators of weibull shape parameter in censored samples. *IEEE Transactions on Reliability*, 32(R):200–202.

Pandya, M. and Jadav, P. (2010). Bayesian estimation of change point in mixture of left truncated exponential and degenerate distribution. *Communication in statistics - Theory and Methods*, 39(15):2725–2742.

Parsian, A. and Kirmani, S. N. U. A. (2002). Estimation under linex loss function, in: A. ullah, a. t. k. wan, a. chaturvedi & m. dekker, eds. *Handbook of Applied Econometrics and Statistical Inference, CRC Press, Boca Raton*, pages 53–76.

Prakash, G. and Singh, D. (2008). Item failure data of weibull failure model under bayesian estimation. *Journal of Statistical Research*, 42(2):131–140.

Prakash, G. and Singh, D. C. (2009). Estimation of the weibull shape parameter in failure censored sampling under the linex loss. *METRON - International Journal of Statistics*, 37(1):31–50.

Singh, D. C., Prakash, G., and Singh, P. (2007). Shrinkage testimators for the shape parameter of pareto distribution using linex loss function. *Communications in Statistics - Theory and Methods*, 36(6):741–753.

Singh, H. P. and Shukla, S. K. (2000). Estimation in the two - parameters weibull distribution with prior information. *IAPQR Transactions*, 25(2):107–118.

Tsionas, E. G. (2002). Bayesian analysis of finite mixtures of weibull distributions. *Communications in Statistics - Theory and Methods*, 31(1):37–48.

Wahed, A. S., Luong, T. M., and Jeong, J. H. (2009). A new generalization of weibull distribution with application to a breast cancer data set. *Statistical Medicine*, 28(16):2077–2094.

Weibull, W. (1951). A statistical distribution functions of wide applicability. *Journal of Applied Mathematics and Mechanics*, 18:293–297.

Whittemore, A. and Altschuler, B. (1976). Lung cancer incidence in cigarette smokes: further analysis of the doll and hill's data for british physicians. *Biometrics*, 32:805–816.