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A Class of Exponential Chain Ratio-Product Type Estimator with Two Auxiliary Variables under Double Sampling Scheme

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ABSTRACT

In this paper an exponential chain ratio-product type estimator in double sampling has been developed using information on two supplementary characters for estimating the finite population mean. The optimum property of the suggested strategy has been studied. Comparisons of the efficiency of the proposed estimator under the optimal condition with other estimators have been presented through empirical investigations.

keywords: Auxiliary information, Exponential chain ratio-product type estimator in double sampling, Mean square error, Efficiency.

1 Introduction

Information on variables correlated with the main variable under study is popularly known as auxiliary information. The use of ratio, regression and product strategies in survey sampling solely depend upon the knowledge of population mean $\bar{X}$ of the auxiliary character x . When this sort of knowledge is lacking, the two-phase (or double) sampling design is adopted to estimate $\bar{X}$ by the sample mean $\bar{x}_1$ of a preliminary large sample

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on which only x is observed. More often, information on another additional auxiliary character is known which is relatively cheaper and less correlated to the main character in comparison to the main auxiliary character x. In such a situation, this information may be used to get more efficient estimators of $\bar{X}$ than x in the preliminary sample.

Swain (1970), Chand (1975); and Sukhatme and Chand (1977) proposed a technique of chaining, the available information on auxiliary characteristic with the main characteristic.

This paper aims at developing a class of chain type estimator with two auxiliary variables. The proposed class is based on exponential chain ratio and product-type estimators suggested by Singh and Choudhury (2012). The proposed estimator in its optimum condition is as efficient as the regression estimator and superior to some other estimators under certain conditions in double sampling design. Numerical illustrations are given in support of the present study.

2 Notations

Let a finite population consists of N distinct identifiable units $U_i (i = 1, 2, 3, ..., N)$. Let y and x denote the study and auxiliary variate taking the values y_i and x_i respectively on the U_i units.

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ be the population means of the study variate y and the auxiliary variate x respectively.

$\bar{Z}$ be the population mean of another auxiliary variate Z which is closely related to X but as compared to X remotely related to Y.

$\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ be the sample mean of size n_1 based on the first phase sample.

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the sample means of variables y and x respectively obtained from the second phase sample of size n.

$\bar{z}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} z_i$ be the sample mean of Z of size n_1.

$C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}}$ and $C_z = \frac{S_z}{\bar{Z}}$ are the coefficients of variation of the study variate y, auxiliary variates x and z respectively.

$\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $\rho_{yz} = \frac{S_{yz}}{S_y S_z}$ and $\rho_{zx} = \frac{S_{zx}}{S_z S_x}$ are the correlation coefficients between y and x, y and z; and x and z respectively.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^2$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$ and $S_z^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(z_i - \overline{Z}\right)^2$ are the population variances of study variate y, auxiliary variates x and z respectively.

$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)\left(x_i - \bar{X}\right)$, $S_{yz} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)\left(z_i - \bar{Z}\right)$ and $S_{zx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(z_i - \overline{Z}\right)\left(x_i - \bar{X}\right)$ are the co-variances between y and x, y and z; and z and x respectively; and

$f = \frac{n}{N}$, $f_1 = \frac{n_1}{N}$, $C_{yx} = \frac{\rho_{yx} C_y}{C_x}$, $C_{yz} = \frac{\rho_{yz} C_y}{C_z}$ and $C_{zx} = \frac{\rho_{zx} C_x}{C_z}$.

3 The Proposed Class of Estimator

Let a first-phase large sample of size n_1 units is drawn from population U_i following simple random sampling without replacement (SRSWOR) scheme, while in the second-phase; a subsample of size $n(n_1 > n)$ is drawn by SRSWOR scheme from n_1 units. We assume that $\rho_{yx} > \rho_{yz} > 0$.

Singh and Choudhury (2012) suggested an exponential chain ratio and product-type estimators for $\bar{Y}$ in double sampling respectively as

$\bar{Y}_{Re}^{dc} = \bar{y}\ exp\left(\frac{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}-\bar{x}}{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}+\bar{x}}\right)$ and $\bar{Y}_{Pe}^{dc} = \bar{y}\ exp\left(\frac{\bar{x}-\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}{\bar{x}+\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}\right)$.

The proposed class of estimator is defined as

$$\bar{Y}_{RPe}^{dc} = \bar{y}\left\{\alpha\ exp\left(\frac{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}-\bar{x}}{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}+\bar{x}}\right) + \beta\ exp\left(\frac{\bar{x}-\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}{\bar{x}+\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}\right)\right\} \tag{1}$$

where α and β are unknown constants such that $\alpha + \beta = 1$.
To obtain the bias(B) and mean square error (M) of the estimator $\bar{Y}_{RPe}^{dc}$, we write $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$, $e'_1 = \frac{\bar{x}_1-\bar{X}}{\bar{X}}$ and $e_2 = \frac{\bar{z}_1-\bar{Z}}{\bar{Z}}$ such that

$$\begin{cases} E(e_0) = E(e_1) = E(e'_1) = E(e_2) = 0,\ E(e_0^2) = \frac{1-f}{n}C_y^2, \\ E(e_1^2) = \frac{1-f}{n}C_x^2,\ E(e_1'^2) = \frac{1-f_1}{n_1}C_x^2,\ E(e_2^2) = \frac{1-f_1}{n_1}C_z^2, \\ E(e_0e_1) = \frac{1-f}{n}C_{yx}C_x^2,\ E(e_0e'_1) = \frac{1-f_1}{n_1}C_{yx}C_x^2,\ E(e_0e_2) = \frac{1-f_1}{n_1}C_{yz}C_z^2, \\ E(e_1e'_1) = \frac{1-f_1}{n_1}C_x^2,\ E(e_1e_2) = \frac{1-f_1}{n_1}C_{zx}C_z^2,\ E(e'_1e_2) = \frac{1-f_1}{n_1}C_{zx}C_z^2. \end{cases} \tag{2}$$

Expressing $\bar{Y}_{RPe}^{dc}$ in terms of e's and retaining terms up to second powers of e's, we have

$$\begin{aligned} \bar{Y}_{RPe}^{dc} - \bar{Y} \cong\ & \bar{Y}\Bigg[e_0 + \frac{1}{2}\left(e_1 + e_2 - e'_1 + e_0e_1 + e_0e_2 - e_0e'_1\right) - \frac{1}{8}\left(e_1^2 + e_2^2\right) + \frac{3}{8}e_1'^2 \\ & + \frac{1}{4}\left(e_1e_2 - e_2e'_1 - e_1e'_1\right) \\ & + \alpha\left\{e'_1 - e_1 - e_2 + e_0e'_1 - e_0e_1 - e_0e_2 - \frac{1}{2}\left(e_1'^2 - e_1^2 - e_2^2\right)\right\}\Bigg] \end{aligned} \tag{3}$$

Therefore, the bias of the estimator $\bar{Y}_{RPe}^{dc}$ can be obtained by using the results of (2) in equation (3) as

$B\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}\left\{\left(\frac{1}{2}-\alpha\right)\left(\frac{1-f^*}{n}C_{yx}C_x^2 + \frac{1-f_1}{n_1}C_{yz}C_z^2\right) + \left(\frac{1}{2}\alpha - \frac{1}{8}\right)\left(\frac{1-f^*}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right)\right\}$,
where $f^* = \frac{n}{n_1}$.

From equation (3), we have

$$\bar{Y}_{RPe}^{dc} - \bar{Y} \cong \bar{Y}\left\{e_0 + \left(\frac{1}{2} - \alpha\right)\left(e_1 + e_2 - e'_1\right)\right\} \tag{4}$$

Squaring both sides in equation (4), taking expectations and using the results of (2), we get the MSE of $\bar{Y}_{RPe}^{dc}$ to the first degree of approximation as

$$M\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 + (1-2\alpha)\left(\frac{1-f^*}{n} C_{yx} C_x^2 + \frac{1-f_1}{n_1} C_{yz} C_z^2\right) + \left(\frac{1}{2}-\alpha\right)^2 \times \left(\frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2\right)\right\} \tag{5}$$

Minimization of equation (5) with respect to α yields its optimum value as

$$\alpha_{opt.} = \frac{1}{2} + \frac{\frac{1-f^*}{n} C_{yx} C_x^2 + \frac{1-f_1}{n_1} C_{yz} C_z^2}{\frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2} \tag{6}$$

Substituting the value of $\alpha_{opt.}$ from equation (6) in equation (5), we get the minimum value of $M\left(\bar{Y}_{RPe}^{dc}\right)$ as

$$min.M\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 - \frac{\left(\frac{1-f^*}{n} C_{yx} C_x^2 + \frac{1-f_1}{n_1} C_{yz} C_z^2\right)^2}{\frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2} \right\}.$$

Remark 1:

For $(\alpha, \beta) = (1, 0)$, the proposed class of estimator reffered to equation (1) reduces to the 'exponential chain ratio-type estimator in double sampling' suggested by Singh and Choudhury (2012) as

$$\bar{Y}_{Re}^{dc} = \bar{y} \exp\left(\frac{\bar{x}_1 \frac{\bar{Z}}{\bar{z}_1} - \bar{x}}{\bar{x}_1 \frac{\bar{Z}}{\bar{z}_1} + \bar{x}}\right)$$

and the MSE of $\bar{Y}_{Re}^{dc}$ can be obtain by putting $\alpha = 1$ in equation (5) as

$$M\left(\bar{Y}_{Re}^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2\right) - \frac{1-f^*}{n} C_{yx} C_x^2 - \frac{1-f_1}{n_1} C_{yz} C_z^2 \right\} \tag{7}$$

while $(\alpha, \beta) = (0, 1)$, the proposed class of estimator is reduces to the 'exponential chain product-type estimator in double sampling' suggested by Singh and Choudhury (2012) as

$$\bar{Y}_{Pe}^{dc} = \bar{y} \exp\left(\frac{\bar{x} - \bar{x}_1 \frac{\bar{Z}}{\bar{z}_1}}{\bar{x} + \bar{x}_1 \frac{\bar{Z}}{\bar{z}_1}}\right)$$

and the MSE of $\bar{Y}_{Pe}^{dc}$ can be obtain by putting $\alpha = 0$ in equation (5) as

$$M\left(\bar{Y}_{Pe}^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2\right) + \frac{1-f^*}{n} C_{yx} C_x^2 + \frac{1-f_1}{n_1} C_{yz} C_z^2 \right\} \tag{8}$$

4 Efficiency Comparisons

4.1 with exponential chain ratio-type estimator in double sampling, Singh and Choudhury (2012)

From equations (5) and (7), we have

$M\left(\bar{Y}_{Re}^{dc}\right) - M\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}^2 (1-\alpha)(\alpha A - 2B)$,

where $A = \frac{1-f^*}{n} C_x^2 + \frac{1-f_1}{n_1} C_z^2$ and $B = \frac{1-f^*}{n} C_{yx} C_x^2 + \frac{1-f_1}{n_1} C_{yz} C_z^2$.

Therefore, the proposed estimator is better than exponential chain ratio-type estimator in double sampling if

either, $\frac{2B}{A} < \alpha < 1$. or, $1 < \alpha < \frac{2B}{A}$.

or equivalently, $\min\left(1, \frac{2B}{A}\right) < \alpha < \max\left(1, \frac{2B}{A}\right)$.

4.2 with exponential chain product-type estimator in double sampling, Singh and Choudhury (2012)

From equations (5) and (8), we have

$M\left(\bar{Y}_{Pe}^{dc}\right) - M\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}^2 \alpha \left\{(1-\alpha) A + 2B\right\}$.

Therefore, the proposed estimator is more efficient than $\bar{Y}_{Pe}^{dc}$ if

either, $1 + \frac{2B}{A} < \alpha < 0$. or, $0 < \alpha < 1 + \frac{2B}{A}$.

or equivalently, $\min\left(0, 1 + \frac{2B}{A}\right) < \alpha < \max\left(0, 1 + \frac{2B}{A}\right)$.

4.3 with chain ratio estimator in double sampling, Chand (1975)

The MSE of chain ratio estimator in double sampling '$\bar{Y}_R^{dc} = \bar{y} \frac{\bar{x}_1}{\bar{x}} \frac{\bar{Z}}{\bar{z}_1}$' is

$$M\left(\bar{Y}_R^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 + \frac{1-f^*}{n} C_x^2 (1 - 2C_{yx}) + \frac{1-f_1}{n_1} C_z^2 (1 - 2C_{yz}) \right\} \tag{9}$$

From equations (5) and (9), we have

$M\left(\bar{Y}_R^{dc}\right) - M\left(\bar{Y}_{RPe}^{dc}\right) = \bar{Y}^2 (1.5 - \alpha) \left\{(0.5 + \alpha) A - 2B\right\}$.

Therefore, the proposed estimator is more efficient than chain ratio estimator in double sampling if

either, $-0.5 + \frac{2B}{A} < \alpha < 1.5$. or, $1.5 < \alpha < -0.5 + \frac{2B}{A}$.

or equivalently, $\min\left(1.5, -0.5 + \frac{2B}{A}\right) < \alpha < \max\left(1.5, -0.5 + \frac{2B}{A}\right)$.

4.4 with chain product estimator in double sampling

The MSE of chain product estimator in double sampling '$\bar{Y}_P^{dc} = \bar{y} \frac{\bar{x}}{\bar{x}_1} \frac{\bar{z}_1}{\bar{Z}}$' is

$$M\left(\bar{Y}_P^{dc}\right) = \bar{Y}^2 \left\{ \frac{1-f}{n} C_y^2 + \frac{1-f^*}{n} C_x^2 (1 + 2C_{yx}) + \frac{1-f_1}{n_1} C_z^2 (1 + 2C_{yz}) \right\} \tag{10}$$

From equations (5) and (10), we have

$M\left(\bar{Y}_{P}^{dc}\right)-M\left(\bar{Y}_{RPe}^{dc}\right)=\bar{Y}^{2}\left(0.5+\alpha\right)\left\{\left(1.5-\alpha\right)A+2B\right\}.$

Therefore, the proposed estimator is better than chain product estimator in double sampling if

either, $-0.5<\alpha<1.5+\frac{2B}{A}$. or, $1.5+\frac{2B}{A}<\alpha<-0.5$.

or equivalently, $\min\left(-0.5,1.5+\frac{2B}{A}\right)<\alpha<\max\left(-0.5,1.5+\frac{2B}{A}\right).$

4.5 with sample mean per unit estimator $\bar{y}$

The MSE of sample mean per unit estimator $\bar{y}$ is

$$M\left(\bar{y}\right)=\bar{Y}^{2}\frac{1-f}{n}C_{y}^{2} \tag{11}$$

From equations (5) and (11), we have

$M\left(\bar{y}\right)-M\left(\bar{Y}_{RPe}^{dc}\right)=\bar{Y}^{2}\left(0.5-\alpha\right)\left\{-2B-\left(0.5-\alpha\right)A\right\}.$

Therefore, the proposed estimator is more efficient than $\bar{y}$ if

either, $0.5<\alpha<0.5+\frac{2B}{A}$. or, $0.5+\frac{2B}{A}<\alpha<0.5$.

or equivalently, $\min\left(0.5,0.5+\frac{2B}{A}\right)<\alpha<\max\left(0.5,0.5+\frac{2B}{A}\right).$

5 Empirical Study

To examine the merits of the proposed estimator, we have considered five natural population data sets. The sources of populations, nature of the variates y, x and z; and the values of the various parameters are given as.

Population I -Source: Murthy (1967)

Y: Area under wheat in 1964, X: Area under wheat in 1963, Z: Cultivated area in 1961.

N=34, n=7, n_1=10, $\bar{Y}$=199.44 acre, $\bar{X}$=208.89 acre, $\bar{Z}$=747.59 acre, ρ_{yx}=0.9801, ρ_{yz}=0.9043, ρ_{zx}=0.9097, C_y^2=0.5673, C_x^2=0.5191, C_z^2=0.3527.

Population II -Source: Cochran (1977)

Y: Number of 'Placebo' children, X: Number of paralytic polio cases in the placebo group, Z: Number of paralytic polio cases in the 'not inoculated' group.

N=34, n=10, n_1=15, $\bar{Y}$=4.92, $\bar{X}$=2.59, $\bar{Z}$=2.91, ρ_{yx}=0.7326, ρ_{yz}=0.6430, ρ_{zx}=0.6837, C_y^2=1.0248, C_x^2=1.5175, C_z^2=1.1492.

Population III-Source: Sukhatme and Chand (1977)

Y: Apple trees of bearing age in 1964, X: Bushels of apples harvested in 1964, Z: Bushels of apples harvested in 1959.

N=200, n=20, n_1=30, $\bar{Y} = 0.103182 \times 10^4$, $\bar{X} = 0.293458 \times 10^4$, $\bar{Z} = 0.365149 \times 10^4$, ρ_{yx}=0.93, ρ_{yz}=0.77, ρ_{zx}=0.84, C_y^2=2.55280, C_x^2=4.02504, C_z^2=2.09379.

Population IV-Source: Srivnstava et al. (1989)

Y: The measurement of weight of children , X: Mid arm circumference of children, Z: Skull circumference of children.

N=82, n=25, n_1=43, $\bar{Y}$=5.60 kg, $\bar{X}$=11.90 cm, $\bar{Z}$=39.80 cm, ρ_{yx}=0.09, ρ_{yz}=0.12, ρ_{zx}=0.86, C_y^2=0.0107, C_x^2=0.0052, C_z^2=0.0008.

Population V-Source: Srivnstava et al. (1989)

Y: The measurement of weight of children , X: Mid arm circumference of children, Z: Skull circumference of children.

N=55, n=18, n_1=30, $\bar{Y}$=17.08 kg, $\bar{X}$=16.92 cm, $\bar{Z}$=50.44 cm, ρ_{yx}=0.54, ρ_{yz}=0.51, ρ_{zx}=-0.08, C_y^2=0.0161, C_x^2=0.0049, C_z^2=0.0007.

To reflect the gain in the efficiency of the proposed estimator $\bar{Y}_{RPe}^{dc}$ over the estimators $\bar{Y}_{Re}^{dc}$, $\bar{Y}_{Pe}^{dc}$, $\bar{Y}_{R}^{dc}$, $\bar{Y}_{P}^{dc}$ and $\bar{y}$, the effective ranges of α along with its optimum values are presented in Table 1 with respect to the population data sets.

Table 1: Effective ranges of α and its optimum values of the estimator $\bar{Y}_{RPe}^{dc}$

	Ranges of α under which the estimator $\bar{Y}_{RPe}^{dc}$ is better than					Opt. values
Pop. ↓	$\bar{Y}_{Re}^{dc}$	$\bar{Y}_{Pe}^{dc}$	$\bar{Y}_{R}^{dc}$	$\bar{Y}_{P}^{dc}$	$\bar{y}$	α_{opt}
I	(1.00, 2.18)	(0.00, 3.18)	(1.50, 1.68)	(-0.50, 3.68)	(0.50, 2.68)	1.5892
II	(1.00, 1.21)	(0.00, 2.21)	(0.71, 1.50)	(-0.50, 2.71)	(0.50, 1.71)	1.1044
III	(1.00, 1.58)	(0.00, 2.58)	(1.08, 1.50)	(-0.50, 3.08)	(0.50, 2.08)	1.2921
IV	(0.32, 1.00)	(0.00, 1.32)	(-0.18, 1.50)	(-0.50, 1.82)	(0.50, 0.82)	0.6577
V	(1.00, 2.22)	(0.00, 3.22)	(1.50, 1.72)	(-0.50, 3.72)	(0.50, 2.72)	1.6090

To observe the relative performance of different estimators of $\bar{Y}$, we have computed the percentage relative efficiencies of the proposed estimator $\bar{Y}_{RPe}^{dc}$, exponential chain ratio and product-type estimators $\left(\bar{Y}_{Re}^{dc}, \bar{Y}_{Pe}^{dc}\right)$, chain ratio and chain product estimators $\left(\bar{Y}_{R}^{dc}, \bar{Y}_{P}^{dc}\right)$ in double sampling and sample mean per unit estimator $\bar{y}$ with respect to usual unbiased estimator $\bar{y}$. The findings are presented in Table 2.

Table 2: Percentage relative efficiencies of different estimators w.r.t. $\bar{y}$.

Estimators→	$\bar{y}$	$\bar{Y}_R^{dc}$	$\bar{Y}_P^{dc}$	$\bar{Y}_{Re}^{dc}$	$\bar{Y}_{Pe}^{dc}$	$\bar{Y}_{RPe}^{dc}$
Population I	100.00	730.78	30.05	259.54	50.48	763.27
Population II	100.00	136.91	25.96	184.36	47.55	189.27
Population III	100.00	279.93	26.02	247.82	46.58	322.94
Population IV	100.00	81.92	70.22	97.11	88.38	100.81
Population V	100.00	131.91	61.01	120.57	78.75	132.32

6 Conclusions

On the basis of the theoretical and empirical results derived in this paper, we may conclude it as follows

i. Table 1 shows that there is a wide scope of choosing α for which the proposed class of estimator '$\bar{Y}_{RPe}^{dc}$' performs better than other estimators.

ii. From Table 2, it is evident that the proposed class of estimator '$\bar{Y}_{RPe}^{dc}$' is more efficient than all other estimators $\bar{Y}_{Re}^{dc}$, $\bar{Y}_{Pe}^{dc}$, $\bar{Y}_R^{dc}$, $\bar{Y}_P^{dc}$ and $\bar{y}$ with considerable gain in efficiency.

Thus, the use of the proposed class of estimator is preferable over other estimators.

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