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ON SYSTEM RELIABILITY FOR MULTI-COMPONENT HALF NORMAL LIFE TIME

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Abstract: *The stress–strength models have been widely used for reliability design of systems. In these models the reliability is defined as the probability that the strength is larger than the stress. If the strength of the component is sufficient to withstand the stress, then the component is functional. Otherwise, the component fails immediately. This paper considers the problem of strength of a manufactured item against an array of stresses following half-normal distribution.*

Keywords: *Strength reliability, product reliability, stress of materials, half-normal lifetime, power function distribution.*

1. Introduction

With the pervasive use of engineering systems in modern society and people's reliance on them in daily life, work, and societal functions, we need to make sure that these systems meet people's expectations for quality and reliability. In modern industrial societies, new products appear on the market at an ever increasing pace. This is due to rapid advances in technology and constantly increasing demands from customers, with each as a driver to the other. As a result, products are getting more complex to design and build and the performance capabilities are increasing with each new product generation.

In today's competitive global market, the Reliability Engineering practice of stress-strength testing has an interesting history. The statistical treatment based on two basic variables X and Y, the first measuring the random stress applied to a particular item on test and the second measuring the strength of the item, generally interpreted as the maximal amount of stress the item can tolerate i.e. its breaking strength or its maximal capacity. The strength-reliability of an

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item i.e. its chance of surviving the stress to which it is subjected, is thus captured through the $P(Y > X)$, where X and Y are taken to be independent continuous random variables. For a comprehensive treatment of stress-strength models, see [5]. [1, 2] have studied stress-strength problem from a different angle: Assuming the form of the distributions of X and Y to be known and find the parameters of the distribution so that the desired level of strength-reliability is achieved. For this purpose, they assumed exponential/power function strength and exponential stress respectively. Recently, [3] obtained the strength-reliability for multi-component stress model when each component exponentially distributed. Following this, if the stress of each component distributed half-normally then we obtain the strength-reliability of an item for multi-component stress model following half normal distribution.

2. System reliability for multi-component stress

The choice of the stress distribution with an infinite range may be justified, as it is genuinely possible to face a very huge stress that may be regarded as tending to infinity.
Let the stress variables $(X_i; i = 1, 2, 3, ..., n)$ be independent and identically distributed (iid) as half normal with probability density function:

$$f(x) = \frac{2}{\pi\theta} e^{-x^2\theta^2/\pi}, \qquad x > 0, \;\; \theta > 0 \tag{1}$$

In general the combined effect of *n*-stresses is given by:

$$f(z) = \frac{2\sqrt{n}}{\pi\theta} e^{-n z^2\theta^2/\pi}, \qquad z = (x_1 + x_2 + ... + x_n) > 0, \;\; \theta > 0 \tag{2}$$

which is again half normal life time distribution.
However, the design strength of an equipment should only be limited to a finite range. This is so because the strength of an engineering product is always a function of a combination of a set of subcomponent and as we know that the strength of a chain lies in its weakest link, not all the subcomponents are likely to have an infinite strength.
Let strength Y follows power function distribution with *pdf*, given by:

$$g(y) = \left(\frac{m}{\beta}\right)\left(\frac{y}{\beta}\right)^{m-1}, \qquad 0 < y < \beta, \; m > 0 \tag{3}$$

where β is scale parameter and m is shape parameter.
Since the maximum possible strength of y is β , y cannot exceed z if z exceeds β. $P(Z > \beta)$ gives the disaster information.

By definition:

$$\alpha = P(Z > \beta) = \int_{\beta}^{\infty} \frac{2\sqrt{n}}{\pi\theta}\, e^{-(n z^2 / \pi\theta^2)}\, dz$$

$$= 1 - \frac{1}{\sqrt{\pi}} \int_{0}^{n\beta^2/\pi\theta^2} t^{-1/2}\, e^{-t}\, dt$$

For a fixed β and known θ, if we let, $r = \frac{\beta}{\theta}$. Then:

$$\alpha = 1 - \frac{1}{\sqrt{\pi}} \int_{0}^{nr^2/\pi} t^{-1/2}\, e^{-t}\, dt$$

In practice one cannot avert a disaster. Naturally, we would fix this α to be as small as possible. Choose the value of r in such a way that the probability of disaster is pre-determined value of α, the tolerance level of the item.

$$I\left(n r^2 \sqrt{2}/\pi,\ \ 1/2\right) = 1 - \alpha \qquad (4)$$

For known values of α, we can approximate the values of r from the incomplete gamma table (Pearson, 1957).
Finally, our aim is to find the strength-reliability of the item under the stress and strength having density functions given in (3) and (4). Using the result of [6], the strength reliability of an item is given by:

$$R = \int_{0}^{r\theta} \int_{1}^{r\theta/z} z\, \frac{2\sqrt{n}}{\pi\theta} e^{-(nz^2/\pi\theta^2)}\, \frac{m}{(r\theta)^m} (vz)^{m-1}\, dv\, dz`$$

$$= \frac{2\sqrt{n}}{\pi\theta} \frac{m}{(r\theta)^m} \int_{0}^{r\theta} z^m\, e^{-(nz^2/\pi\theta^2)}\, \frac{1}{m} \left[\left(\frac{r\theta}{z} \right)^m - 1 \right] dz$$

$$= \frac{2\sqrt{n}}{\pi\theta} \int_{0}^{r\theta} e^{-(nz^2/\pi\theta^2)}\, dz \; - \; \frac{2\sqrt{n}}{\pi\theta} \frac{1}{(r\theta)^m} \int_{0}^{r\theta} z^m\, e^{-(nz^2/\pi\theta^2)}\, dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{0}^{nr^2/\pi} t^{-1/2}\, e^{-t}\, dt \; - \; \frac{1}{\sqrt{\pi}} \frac{1}{(r\sqrt{n}/\sqrt{\pi})^m} \int_{0}^{nr^2/\pi} (t^{1/2})^{m-1}\, e^{-t}\, dt, \qquad (5)$$

where $t = (nz^2 / \pi\theta^2)$.

The expression (5) is of the form of incomplete gamma function. So it is not easy to calculate R for given values of m, n and r. However, it can be approximated by using Table of incomplete gamma function [7] or by appropriate statistical package. We obtain R for selected values of m and r as shown in Tables given below.

Table 1. Strength-reliability of an item for selected values of r and m for n=4.

$r \rightarrow$ $m \downarrow$	**1.50**	**1.75**	**2.0**	**2.25**	**2.50**	**2.75**	**3.0**	**3.5**	**4.0**
2.0	0.8306	0.8730	0.9020	0.9224	0.9372	0.9481	0.9563	0.9679	0.9754
3.0	0.8925	0.9287	0.9513	0.9656	0.9749	0.9811	0.9854	0.9908	0.9939
4.0	0.9224	0.9537	0.9717	0.9820	0.9882	0.9919	0.9943	0.9969	0.9982
6.0	0.9491	0.9742	0.9869	0.9932	0.9963	0.9979	0.9988	0.9995	0.9998
8.0	0.9605	0.9821	0.9921	0.9965	0.9984	0.9992	0.9996	0.9999	1.0000
10.0	0.9666	0.9859	0.9944	0.9978	0.9991	0.9997	0.9999	1.0000	1.0000
12.0	0.9702	0.9881	0.9956	0.9984	0.9995	0.9998	0.9999	1.0000	1.0000

Table 2. Strength-reliability of an item for selected values of r and m for n=5.

$r \rightarrow$ $m \downarrow$	**1.50**	**1.75**	**2.0**	**2.25**	**2.50**	**2.75**	**3.0**	**3.5**	**4.0**
2.0	0.8622	0.8977	0.9215	0.9379	0.9497	0.9584	0.9651	0.9743	0.9804
3.0	0.9199	0.9481	0.9649	0.9753	0.9820	0.9865	0.9896	0.9934	0.9956
4.0	0.9462	0.9692	0.9816	0.9885	0.9924	0.9948	0.9963	0.9980	0.9988
6.0	0.9685	0.9853	0.9929	0.9964	0.9981	0.9989	0.9994	0.9997	0.9999
8.0	0.9774	0.9909	0.9963	0.9985	0.9993	0.9997	0.9998	1.0000	1.0000
10.0	0.9818	0.9934	0.9977	0.9992	0.9997	0.9999	1.0000	1.0000	1.0000
12.0	0.9843	0.9947	0.9983	0.9995	0.9998	0.9999	1.0000	1.0000	1.0000

3. Discussion and example

An effective reliability engineering planning and management strategy can contribute greatly to successful product design and development through its impact on the ability of the design/development team to meet desired reliability goals on time and within specified cost to the customer. It is a proven fact that all these technical systems are producible, in other words: One can at least make them work at the time of first use. A higher order requirement, however, is that they remain serviceable throughout their expected useful life; i.e. that they are reliable. The consequences of an unreliable functioning of these systems may vary from inconvenience, extra costs, and environmental damage, to even death. Suppose the strength of an item follows power function distribution then it is likely that the possible values of β may have an upper limit, say β_0, as we know that the capacity of accelerating an engine must be subjected to a maximum possible speed. For a fixed tolerance level α, suppose β_α is the desired value of β. In case

$\beta_\alpha \le \beta_0$ and known n, we may obtain the required value of m say m_α, by using the above tables, so that the manufactured item have the parameters (m_α, β_α) and the desired level of strength-reliability is achieved. However, if $\beta_\alpha > \beta_0$, we adjust the parameter α, or go for an alternate item. For example, suppose $\theta = 100$, strength-reliability of 0.95 or more can be obtained for different combination of m and r, keeping in mind the possible extensions of m, β and for known n, we may find the optimal combination.

Let us consider an example, if a system with five components in series has a reliability objective of 95% for a given operating time, the uniform allocation of the objective to all components could require each component to have a reliability of 99% for the specified operating time, since $0.99^5 \cong 0.95$. While this manner of allocation is easy for calculations, it is generally not the best way to allocate reliability for a system facing stress(s). The optimum method of allocating reliability in that case would be to take into account the cost or relative difficulty of improving the reliability of different subsystems or components. Clearly, in such cases the above table(s) may be useful, if available, otherwise the result (5) may be utilized for particular cases and known values of n. For example, the above example corresponds to $n = 5$, Table 2, is therefore the relevant table. Even for the simple case of $m = 2$, we get $r = 2.50$ approximately, which implies that the components may face 2.5 times average stress as that of the single stress so as to have a 95% reliability against it. If $n = 1$, i.e., single stress is working against strength, as obtained by [4].

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