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MEAN VALUES OF LOCATION AND DISPERSION STATISTICS IN A TWO-WAY TABLE: A COMPARISON AMONG DIFFERENT SCORES SYSTEMS

Rosario Oliviero*

Facoltà di Medicina e Chirurgia, Seconda Università degli Studi di Napoli, Italy

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Abstract. *The main object of this paper is to verify that, in a two-way contingency table, the mean values, of location and dispersion statistics, do not depend significantly on the particular score system. More in detail, we compare Yates scores, inverse distributions based scores, Nair scores, binary ensemble mid-rank and Apache II scores. We perform our analysis by using a Monte Carlo procedure.*

Keywords: *Dispersion effects, location effects, Monte Carlo methods, two-way contingency tables.*

1. Introduction

Correspondence analysis, in most general meaning, is a fundamental tool in order to graphically identify the nature of the association between categorical variables of a contingency table. In particular, as one of the categorization is ordered, it is possible to perform a more detailed analysis than usual chi-squared statistic, by using this methodology. In fact, usual Pearson's test may not be significant.

In this contest, a fundamental contribution is given by the achievements of Best and Rayner (1998). Their procedure involves a partition of chi-squared statistic in terms which represent location and dispersion effects for the whole contingency table. By using this approach, one can perform a partition on the location statistic so defined, in order to evaluate the important factors. Moreover, location and dispersion components are the basis for weakly optimal directional tests and complementary to Pearson's test (Rayner and Best, 2000). The method appears relatively simple if compared to other recent correspondent techniques, both conceptually and numerically.

* E-mail: rosario.oliviero@virgilio.it

In particular, location and dispersion components, like residual effects, may be expressed by using orthogonal Emerson polynomials (Beh et al., 2008). The polynomials' variables are scores assigned to every column of a contingency table; alternatively, we may utilize Nair (1986) scores, as stated by Best et al. (1998), or Rayner and Best (2000). The problem of selecting the scores is still open in contemporary applied statistic literature. In this work we show, by performing Monte Carlo simulations, that, in most cases, the choice among typical scores systems does not influence the mean values of both location and dispersion statistics.

2. Pearson statistics and multinomial components

Consider a r by c contingency table with counts N_{ij}, row marginals $N_{i\bullet} = N_{i1} +...+ N_{ic}$, column marginals $N_{\bullet j} = N_{1j} +...+ N_{rj}$ ($i \in \{1, 2,..., r\}$, $j \in \{1, 2,..., c\}$) and total counts $n = N_{\bullet 1} +...+ N_{\bullet c} = N_{1\bullet}+...+ N_{r\bullet}$. Let the (i, j)'th cell of relative frequencies be denoted by $p_{ij} = N_{ij}/n$; let $p_{i\bullet} = N_{i\bullet}/n$ and $p_{\bullet j} = N_{\bullet j}/n$ be, respectively, the i-th row and j-th column marginal proportion. Finally, suppose that scores $s(1)$, $s(2)$,..., $s(c)$ are assigned to classes 1, 2,..., c, respectively. The usual Pearson chi-squared statistic is:

$$X_p^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(N_{ij}-E_{ij}\right)^2}{E_{ij}}, \qquad (1)$$

where $E_{ij} = N_{i\bullet}N_{\bullet j}/n$. As stated in Best and Rayner (1998), or Rayner and Best (2000), its components may be defined as:

$$V_{ui} = \sum_{j=1}^{c}\frac{N_{ij}b_u(j)}{\sqrt{N_{i\bullet}}} \qquad (2)$$

($u \in \{1, 2,..., c-1\}$, $i \in \{1, 2,..., r\}$), where $b_1(j)$, $b_2(j)$,..., $b_{c-1}(j)$ ($j \in \{1, 2,..., c\}$) are Emerson (1968) orthogonal polynomials: they can be computed by setting $b_{-1}(j) = 0$ and $b_0(j) = 1$ for any $j \in \{1, 2,..., c\}$. The other elements can be calculated by using the following formula (Beh et al., 2008):

$$b_u(j) = S_u\left[\left(s(j)-T_u\right)b_{u-1}(j) - P_u b_{u-2}(j)\right], \qquad (3)$$

where

$$T_u = \sum_{j=1}^{c} p_{\bullet j}s(j)b_{u-1}(j)^2$$

$$P_u = \sum_{j=1}^{c} p_{\bullet j}s(j)b_{u-1}(j)b_{u-2}(j),$$

$$S_u = \left[\sum_{j=1}^{c} p_{\bullet j} s(j)^2 b_{u-1}(j)^2 - T_u^{\,2} - P_u^{\,2} \right]^{-\frac{1}{2}}.$$

This recurrence relation may be used when equally spaced integer valued scores are used to describe the ordered structure of the column categories. Moreover, if D_j is the diagonal matrix of relative frequencies, with the (j,j) element $p_{\bullet j}$, then the column polynomials are orthogonal with respect to this matrix. In formulas, we have:

$$\sum_{j=1}^{c} p_{\bullet j} b_{j(v)} b_{j(v')} = \begin{cases} 1 & if \quad v = v' \\ 0 & otherwise \end{cases}.$$

The orthogonal Emerson polynomials whose degree is 1 and 2 can be easily obtained by applying the recurrence relation (3). So they can be written as:

$$b_1(j) = \frac{s(j) - m}{\sqrt{m_2}}$$

$$b_2(j) = a\left[(s(j) - m)^2 - \frac{m_3(s(j) - m)}{m_2} - m_2 \right],$$

where

$$m = \sum_{j=1}^{c} s(j) p_{\bullet j}$$

$$m_d = \sum_{j=1}^{c} (s(j) - m)^d p_{\bullet j}$$

$$a = \left(m_4 - \frac{m_3^{\,2}}{m_2} - m_2^{\,2} \right)^{-0.5}$$

($d \in \{1, 2, 3\}$). It can be shown that

$$X_p^{\,2} = \sum_{u=1}^{c-1} \sum_{i=1}^{r} V_{ui}^{\,2}. \qquad (4)$$

Therefore, a measure Q of the overall location effect, for the contingency table, is

$$Q = V_{11}^{\,2} + \ldots + V_{1r}^{\,2}. \qquad (5)$$

We observe that, for any $i \in \{1, 2,..., r\}$, V_{ir} has an asymptotically standard normal distribution with the linear constraint:

$$\sum_{i=1}^{r} V_{1i}\sqrt{N_{i\bullet}} = 0.$$

As a consequence, Q is approximately distributes as a χ^2_{r-1}.
Now, if $c \geq 3$, the dispersion statistic D can be assessed by

$$D = V_{21}^2 + ... + V_{2r}^2. \qquad (6)$$

Usually, it is assumed that, for any $j \in \{1, 2,..., c\}$, the score s_j is equal to j (Yates, 1948). Sometimes other values are possible. Alternatively, following Best et al. (1998), or Rayner and Best (2000), we can formally define Nair location and dispersion scores by setting, for any $j \in \{1, 2,..., c\}$, $b_1(j) = l_j$ and $b_2(j) = d_j$, in equations (2), (5) and (6), where

$$l_j = \frac{t_j - 0.5}{\sum_{k=1}^{c} p_{\bullet k}(t_k - 0.5)^2}, \qquad (7)$$

$$t_j = p_{\bullet 1} + \ldots + p_{\bullet (j-1)} + p_{\bullet j}/2, \qquad (8)$$

$$d_j = \frac{e_j}{\sum_{k=1}^{c} p_{\bullet k} e_k^2}, \qquad (9)$$

and

$$e_j = l_j(l_j - (l_1^3 p_{\bullet 1} + \ldots + l_c^3 p_{\bullet c})) - 1. \qquad (10)$$

Finally, we observe that location and dispersion statistics can be regarded as two functions, $Q(s)$ and $D(s)$, of scores system s.

3. Computer simulations

The aim of our simulation study has been controlling if the mean values, of Q and D, vary as a consequence of a change of score system. In particular, we have considered: Yates scores, normal distribution based scores and Nair scores; we have named them, respectively, with s_1, s_2, s_3 and s_4. More in detail, we have defined:

$$s_1(j) = j$$

$$s_2(j) = \Phi^{-1}\left(\frac{\left(j - \frac{3}{8}\right)}{\left(c + \frac{1}{4}\right)}\right)$$

$$s_3(j) = \Phi^{-1}\left(\frac{j}{(c+1)}\right)$$

($j \in \{1, 2,..., c\}$), where Φ is the cumulative function of normal distribution. The necessity of utilizing Monte Carlo method derives from the fact that it appears hard to use sensitivity analysis. For simplicity, we have simulated only contingency tables with counts following a uniform distribution on [0, 1]. We may observe that, this is not a restrictive hypothesis. In order to show this property, consider two r by c contingency tables A and B, whose counts are denoted by N_{Aij} and $N_{Bij} = \alpha N_{Aij}$, respectively ($i \in \{1, 2,..., r\}$, $j \in \{1, 2,..., c\}$, $\alpha \in \mathbf{R}$). Denote with Q_X and D_X, respectively, the location and dispersion statistics of contingency table X ($X \in \{A, B\}$). In this case, we can easily prove the equalities $Q_B = \alpha Q_A$ and $D_B = \alpha D_B$. This result is valid for any kind of scores system.
Another preliminary observation is that, for any value of r, if $c = 3$, Q and D are exactly the same for any of the first three scores, s_1, s_2 and s_3. In fact, by direct calculations, we have $s_1(1) = 1$, $s_1(2) = 2$, $s_1(3) = 3$, $s_2(1) = -0.8694$, $s_2(2) = 0$, $s_2(3) = 0.8694$, $s_3(1) = -0.6745$, $s_3(2) = 0$ and $s_3(3) = 0.6745$. Therefore, $x_3 - x_2 = x_2 - x_1$ for any of these three scores systems. This means that the scores are of the form $aj + b$ (where a and b are real constants and $j \in \{1, 2,..., c\}$): it follows that the coefficients of Emerson polynomials are the same (because they are invariant with respect to a linear transformation of the scores), and so the location and dispersion effects.
Therefore, we have simulated several sets of two-way contingency tables. Each set consists of 100000 tables having the same number of rows and the same number of columns. Then, for any set of contingency tables, we have calculated the mean values and the variances of Q and D over the 100000 simulations. So we have verified that the two means, of these statistics, do not depend significantly on the choice of the scores. Table 1 shows our results for some value of r and c. This study tell us that the expected values, of the effects, decrease with the increase of the columns number c.
Secondly, we have generated 500000 contingency tables in which both counts and dimensions are randomly distributed: for any generated table, the raw length is comprised between 2 and 10 and the column length is comprised between 3 and 10. Therefore, we have calculated the mean values and the variances of location and dispersion statistics. Table 2 confirms what stated in Table 1. Furthermore, we have repeated the same calculations by utilizing 50000 simulations. Table 2 shows that the results are practically identical to those obtained by simulating 500000 contingency tables. This means that our estimations are reliable.

Table 1. Mean values and variances of Q and D, for contingency tables, whose values are uniformly distributed in [0, 1], for different score systems. The number simulations is equal to 100000 for each of the nine groups.

	Scores	Mean (Q)	Var (Q)	Mean (D)	Var (D)
$r = 6$, $c = 5$	s_1	0.9046	0.2805	0.9198	0.2967
	s_2	0.9057	0.2770	0.9205	0.2982
	s_3	0.9054	0.2781	0.9203	0.2978
	s_4	0.9000	0.2795	0.9113	0.2929
$r = 6$, $c = 6$	s_1	0.8948	0.2837	0.9070	0.2939
	s_2	0.8960	0.2797	0.9090	0.2942
	s_3	0.8956	0.2810	0.9084	0.2941
	s_4	0.8901	0.2823	0.8977	0.2904
$r = 6$, $c = 8$	s_1	0.8819	0.2879	0.8961	0.2950
	s_2	0.8836	0.2840	0.9008	0.2922
	s_3	0.8831	0.2851	0.8996	0.2929
	s_4	0.8782	0.2866	0.8871	0.2912
$r = 8$, $c = 6$	s_1	1.2464	0.3908	1.2612	0.4056
	s_2	1.2478	0.3848	1.2639	0.4060
	s_3	1.2474	0.3866	1.2631	0.4059
	s_4	1.2417	0.3895	1.2507	0.4013
$r = 8$, $c = 8$	s_1	1.2272	0.3992	1.2428	0.4012
	s_2	1.2293	0.3931	1.2471	0.3958
	s_3	1.2287	0.3949	1.2460	0.3971
	s_4	1.2232	0.3974	1.2330	0.3965
$r = 8$, $c = 10$	s_1	1.2159	0.3965	1.2321	0.3997
	s_2	1.2176	0.3910	1.2395	0.3915
	s_3	1.2171	0.3926	1.2376	0.3934
	s_4	1.2125	0.3953	1.2235	0.3955
$r = 10$, $c = 5$	s_1	1.6134	0.4982	1.6311	0.5186
	s_2	1.6146	0.4913	1.6325	0.5217
	s_3	1.6142	0.4935	1.6321	0.5210
	s_4	1.6081	0.4971	1.6209	0.5147
$r = 10$, $c = 8$	s_1	1.5762	0.5114	1.5915	0.5156
	s_2	1.5786	0.5031	1.5962	0.5084
	s_3	1.5779	0.5055	1.5950	0.5102
	s_4	1.5721	0.5098	1.5812	0.5108
$r = 10$, $c = 10$	s_1	1.5652	0.5123	1.5736	0.5108
	s_2	1.5666	0.5042	1.5810	0.4985
	s_3	1.5662	0.5065	1.5791	0.5015
	s_4	1.5617	0.5112	1.5643	0.5068

Table 2. Mean values and variances of *Q* and *D,* for contingency tables whose values are uniformly distributed in [0, 1], when the number of rows and columns are randomly distributed and less or equal to 10.

Numer of simulations	Scores	Mean (*Q*)	Var (*Q*)	Mean (*D*)	Var (*D*)
5000	s_1	0.8864	0.4774	0.9072	0.5002
	s_2	0.8896	0.4734	0.9088	0.5119
	s_3	0.8886	0.4745	0.9088	0.5125
	s_4	0.8825	0.4756	0.9008	0.4965
50000	s_1	0.8967	0.4920	0.9066	0.4994
	s_2	0.8973	0.4867	0.9085	0.4979
	s_3	0.8970	0.4883	0.9081	0.4983
	s_4	0.8929	0.4903	0.9005	0.4955
500000	s_1	0.8965	0.4882	0.9066	0.5005
	s_2	0.8977	0.4847	0.9077	0.4990
	s_3	0.8973	0.4859	0.9075	0.4994
	s_4	0.8928	0.4865	0.9004	0.4963

We may be a comparison among different scores systems also by considering the ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean $D((s_h))$/Mean ($D(s_1)$) ($h \in \{2, 3, 4\}$). In order to confirm the results stated in Tables 1-2, we must verify that every ratio has a value approximately equal to one. Tables 3 shows that the use of ratios is reliable even by performing 5000 simulations. So doing, we can extend our analysis to contingency tables whose dimensionality is higher. In this way, we have also employed additional scores systems: inverse Weibull PDF based scores, binary ensemble mid-rank and Apache II (Knaus et al., 1985) scores. We have indicated them, respectively, with s_5, s_6, and s_7. More in detail, we define these scores as:

$$s_5(j) = F^{-1}\left(\frac{j-0.3}{c+0.4}; \lambda, \mu\right),$$

where $F(*; \lambda, \mu)$ is the Weibull distribution whose shape and scale parameters are, respectively, $\lambda > 0$ and $\mu > 0$;
$s_6(j)$ are mid-rank scores of the r by c contingency table whose counts $I_{ij}(x)$ are so defined (Hamill and Juras, 2006):

$$I_{ij}(x) = \begin{cases} 1 & if \quad N_{ij} \geq x \\ 0 & otherwise \end{cases}$$

($\min\{N_{ij}, i \in \{1, 2, \ldots, r\}, j \in \{1, 2, \ldots, c\}\} < x < \max\{N_{ij}, i \in \{1, 2, \ldots, r\}, j \in \{1, 2, \ldots, c\}\}$);

$$s_7(j) = \left|\frac{c+1}{2} - j\right|$$

($j \in \{1, 2, \ldots, c\}$). Table 4(a and b) and table 5(a and b) confirm that mean values, of location and dispersion statistics, do not depend on score system.

Table 3. Ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean ($D(s_h)$)/Mean ($D(s_1)$) ($h \in \{2, 3, 4\}$), for contingency tables whose values are uniformly distributed in [0, 1], when the number of rows and columns are randomly distributed and less or equal to 10.

Numer of simulations	Scores	Mean ($Q(s_h)$)/Mean ($Q(s_1)$)	Mean ($D(s_h)$)/Mean ($D(s_1)$)
5000	s_2	1,0036	1,0018
	s_3	1,0025	1,0018
	s_4	0,9956	0,9929
50000	s_2	1,0007	1,0021
	s_3	1,0003	1,0017
	s_4	0,9958	0,9933
500000	s_2	1,0013	1,0012
	s_3	1,0009	1,0010
	s_4	0,9959	0,9932

4. Application

To illustrate our results, we analyze a 6 by 6 table (Table 6) from Best and Rayner (1998). The six row of this table are a result of a 2 by 3 cross-classification of urbanization and region, while the column variable describes the response: it is measured on a categorical scale. Finally, Table 7 illustrates location and dispersion statistics of this data set. We observe that, except for the Apache II scores, the values of location and dispersion statistics have the same order of magnitude. This probably occurs because Apache II scores, unlike all others, if regarded as a function of the variable j, are not increasing.

Table 4a. Ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean $D(s_h)$/Mean ($D(s_1)$) ($h \in \{2, 3, 4, 5, 6, 7\}$), for contingency tables whose values are uniformly distributed in [0, 1]. The number simulations is equal to 5000 for each of the three groups.

Table dimensionality	Scores	Mean ($Q(s_h)$)/Mean ($Q(s_1)$)	Mean ($D(s_h)$)/Mean ($D(s_1)$)
r =10	s_2	0.9987	1.0046
c = 20	s_3	0.9984	1.0030
	s_4	0.9985	0.9969
	s_5 $\lambda = \mu = 5$	0.9990	1.0027
	s_6 $x = 0.5$	0.9980	0.9955
	s_6 $x = 0.2$	0.9984	0.9955
	s_7	1.0064	1.0006

Table 4b. Ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean $D(s_h)$/Mean ($D(s_1)$) ($h \in \{2, 3, 4, 5, 6, 7\}$), for contingency tables whose values are uniformly distributed in [0, 1]. The number simulations is equal to 5000 for each of the three groups.

Table dimensionality	Scores	Mean ($Q(s_h)$)/Mean ($Q(s_1)$)	Mean ($D(s_h)$)/Mean ($D(s_1)$)
$r = 20$ $c = 10$	s_2	0.9993	1.0029
	s_3	0.9991	1.0022
	s_4	0.9991	0.9963
	s_5 $\lambda = \mu = 5$	1.0001	1.0003
	s_6 $x = 0.5$	0.9986	0.9965
	s_6 $x = 0.2$	0.9983	0.9958
	s_7	0.9970	1.0097
$r = 20$ $c = 20$	s_2	1.0038	0.9959
	s_3	1.0031	0.9962
	s_4	0.9992	0.9988
	s_5 $\lambda = \mu = 5$	1.0014	1.0041
	s_6 $x = 0.5$	0.9991	0.9976
	s_6 $x = 0.2$	0.9992	0.9975
	s_7	0.9968	0.9863

5. Conclusion

This work has focused on testing the components of Pearson chi-squared statistic for two dimensional contingency tables. We have shown that, for a few common scores systems used, the mean values, of location and dispersion statistics, do not depend on scores used. Our simulation study tells us that this property remains true as table dimensionality increases. Our study enhances the importance of location and dispersion effects. Our method could be used in order to test a scores system utilized: we suggest to reject a particular scores system if the empirical means, of Q and D, do not coincide with those relative to common scores utilized in statistical literature.

Table 5a. Ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean ($D(s_h)$)/Mean ($D(s_1)$) ($h \in \{2, 3, 4, 5, 6, 7\}$), for contingency tables whose values are uniformly distributed in [0, 1]. The number of rows and columns are randomly distributed in a given interval. The number simulations is equal to 5000 for each of the three groups.

Table dimensionality	Scores	Mean ($Q(s_h)$)/Mean ($Q(s_1)$)	Mean ($D(s_h)$)/Mean ($D(s_1)$)
$2 \le r \le 20$ $3 \le c \le 20$	s_2	1.0024	1.0018
	s_3	1.0019	1.0013
	s_4	0.9983	0.9956
	s_5 $\lambda = \mu = 5$	0.9992	1.0000
	s_6 $x = 0.5$	0.9974	0.9955
	s_6 $x = 0.2$	0.9971	0.9939
	s_7	1.0023	0.9983

Table 5b. Ratios Mean ($Q(s_h)$)/Mean ($Q(s_1)$) and Mean ($D(s_h)$)/Mean ($D(s_1)$) ($h \in \{2, 3, 4, 5, 6, 7\}$), for contingency tables whose values are uniformly distributed in [0, 1]. The number of rows and columns are randomly distributed in a given interval. The number simulations is equal to 5000 for each of the three groups.

Table dimensionality	Scores	Mean ($Q(s_h)$)/Mean ($Q(s_1)$)	Mean ($D(s_h)$)/Mean ($D(s_1)$)
$2 \leq r \leq 40$, $3 \leq c \leq 40$	s_2	1.0021	1.0044
	s_3	1.0017	1.0039
	s_4	0.9993	0.9988
	s_5 $\lambda = \mu = 5$	1.0019	1.0006
	s_6 $x = 0.5$	0.9989	0.9981
	s_6 $x = 0.2$	0.9992	0.9979
	s_7	1.0027	0.9914
$2 \leq r \leq 80$, $3 \leq c \leq 80$	s_2	1.0022	0.9950
	s_3	1.0019	0.9952
	s_4	0.9998	0.9994
	s_5 $\lambda = \mu = 5$	1.0018	1.0002
	s_6 $x = 0.5$	0.9996	0.9993
	s_6 $x = 0.2$	0.9997	0.9992
	s_7	0.9983	0.9988

Table 6. Observed counts in various regions for the olive data.

Urbanization	Region	Response					
		–	–	ϕ	+	++	+++
Urban	Mid-west	20	15	12	17	16	28
	North-east	18	17	18	18	6	25
	South-west	12	9	23	21	19	30
Rural	Mid-west	30	22	21	17	8	12
	North-east	23	18	20	18	10	15
	South-west	11	9	26	19	17	24

Table 7. Location and dispersion statistics, calculated for various scoring systems, for the olive data.

Scores	Q	D
s_1	33.6130	5.7814
s_2	32.6576	5.3081
s_3	32.9786	5.4416
s_4	33.5465	5.8249
s_5	33.0732	4.8911
s_6 $x = 10$	33.5093	5.8200
s_6 $x = 20$	33.3623	5.8382
s_7	5.9934	1.4710

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