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ACCEPTANCE SAMPLING PLANS FOR PERCENTILES BASED ON THE INVERSE RAYLEIGH DISTRIBUTION

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Abstract: *In this article, acceptance sampling plans are developed for the inverse Rayleigh distribution percentiles when the life test is truncated at a pre-specified time. The minimum sample size necessary to ensure the specified life percentile is obtained under a given customer's risk. The operating characteristic values (and curves) of the sampling plans as well as the producer's risk are presented. Two examples with real data sets are also given as illustration.*

Keywords: *Acceptance sampling, consumer's risk, operating characteristic function, producer's risk, truncated life tests, producer's risk.*

1. Introduction

The acceptance sampling plans are concerned with accepting or rejecting a submitted lot of a large size of products on the basis of the quality of the products inspected in a sample taken from the lot. An acceptance sampling plan is a specified plan that establishes the minimum sample size to be used for testing. In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. If the quality characteristic is regarding the lifetime of the product, the acceptance sampling problem becomes a life test. Traditionally, when the life test indicates that the mean life of products exceeds the specified one, the lot of products is accepted, otherwise it is rejected. For the purpose of reducing the test time and cost, a truncated life test may be conducted to determine the smallest sample size to ensure a certain mean life of products when the life test is terminated at a pre-assigned time , and the number of failures observed does not exceed a given acceptance number c. The decision is to accept the lot if a pre-determined mean life can be reached with a pre-determined high probability which provides protection to consumers. Therefore, the life test is ended at the time

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the failure is observed or at the pre-assigned time, whichever is earlier. For such a truncated life test and the associated decision rule; we are interested in obtaining the smallest sample size to arrive at a decision. Rosaiah and Kantam [20] developed an acceptance sampling procedure for the inverse Rayleigh distribution mean under a truncated life test. Some other studies regarding truncated life tests can be found in Epstein [3], Sobel and Tischendrof [22], Goode and Kao [5], Gupta and Groll [7], Gupta [6], Fertig and Mann [4], Kantam and Rosaiah [9], Kantam et al. [10], Baklizi [1], Wu and Tsai [25], Rosaiah et al. [21], Tsai and Wu [24], Balakrishnan et al. [2] and Rao et al. ([17], [18] & [19]).

All these authors considered the design of acceptance sampling plans based on the population mean under a truncated life test. Whereas Lio et al. [12] considered acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles and they proposed that the acceptance sampling plans based on mean may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. When the quality of a specified low percentile is concerned, the acceptance sampling plans based on the population mean could pass a lot which has the low percentile below the required standard of consumers. Furthermore, a small decrease in the mean with a simultaneous small increase in the variance can result in a significant downward shift in small percentiles of interest. This means that a lot of products could be accepted due to a small decrease in the mean life after inspection. But the material strengths of products are deteriorated significantly and may not meet the consumer's expectation. Therefore, engineers pay more attention to the percentiles of lifetimes than the mean life in life testing applications. Moreover, most of the employed life distributions are not symmetric. In viewing Marshall and Olkin [13], the mean life may not be adequate to describe the central tendency of the distribution. This reduces the feasibility of acceptance sampling plans if they are developed based on the mean life of products. Actually, percentiles provide more information regarding a life distribution than the mean life does. When the life distribution is symmetric, the 50th percentile or the median is equivalent to the mean life. Hence, developing acceptance sampling plans based on percentiles of a life distribution can be treated as a generalization of developing acceptance sampling plans based on the mean life of items. Balakrishnan et al. [2] proposed the acceptance sampling plans could be used for the quantiles and derived the formulae whereas Lio et al. [12] developed for the acceptance sampling plans for any other percentiles of the Birnbaum-Saunders (BS) model. They have developed the acceptance sampling plans for percentile by replace the scale parameter by the 100qth percentile in the BS distribution function. Rao and Kantam [16] developed acceptance sampling plans from truncated life tests based on the log-logistic distribution for Percentiles. These reasons motivate to develop acceptance sampling plans based on the percentiles of the inverse Rayleigh distribution under a truncated life test.

The rest of the article is organized as follows. The proposed sampling plans are established for the inverse Rayleigh percentiles under a truncated life test, along with the operating characteristic (OC) and some relevant tables are given in Section 2. Two examples based on real fatigue life data sets are provided for the illustration in Section 3, Future work is given in Section 4 and discussion and some conclusions are made in Section 5.

2. Acceptance Sampling Plans

Assume that the lifetime of a product follows an inverse Rayleigh distribution which has the following probability density function (pdf) and cumulative distribution function (cdf), respectively:

$$f(t;\sigma) = \frac{2\sigma^2}{t^3} e^{-(\sigma/t)^2};\ \ t \ge 0, \sigma > 0\,, \tag{1}$$

and

$$F(t;\sigma) = e^{-(\sigma/t)^2};\ \ t \ge 0, \sigma > 0, \tag{2}$$

where σ is the scale parameter. The failure rate of a single parameter inverse Rayleigh distribution is increasing for t<1.0694543σ and decreasing for *t*>1.0694543σ as shown by Mukherjee and Saran [14]. Given $0 < q < 1$ the 100qth percentile (or the qth quantile) is given by:

$$t_q = \sigma\left(-\ln q\right)^{-1/2}. \tag{3}$$

The t_q is increases as q increases. Let $\eta = \left(-\ln q\right)^{-1/2}$. Then, Eq. (3) implies that

$$\sigma = t_q/\eta\,. \tag{4}$$

To develop acceptance sampling plans for the inverse Rayleigh percentiles, the scale parameter σ in the inverse Rayleigh cdf is replaced by Eq. (4) and the inverse Rayleigh cdf is rewritten as:

$$F(t) = e^{-\left((t_q/\eta)/t\right)^2};\quad t > 0.$$

Letting $\delta = t/t_q$, F(t) can be rewritten emphasizing its dependence on δ as:

$$F(t;\delta) = e^{-(1/\eta\delta)^2};\quad t > 0\,.$$

Taking partial derivative with respect to δ, we have:

$$\frac{\partial F(t;\delta)}{\partial\delta} = \frac{2}{\eta\delta^3} e^{-(1/\eta\delta)^2};\quad t > 0\,.$$

A common practice in life testing is to terminate the life test by a pre-determined time t, the probability of rejecting a bad lot be at least p^*, and the maximum number of allowable bad items to accept the lot be c. The acceptance sampling plan for percentiles under a truncated life test is to set up the minimum sample size n for this given acceptance number c such that the consumer's risk, the probability of accepting a bad lot, does not exceed 1- p^*. A bad lot means that the true 100qth percentile, t_q, is below the specified percentile, t_q^0. Thus, the probability p^* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to p^*. Therefore, for a given p^*, the proposed acceptance sampling plan can be characterized by the triplet $(n, c, t/t_q^0)$.

2.1 Minimum Sample Size

For a fixed p^* our sampling plan is characterized by $(n, c, t/t_q^0)$. Here we consider sufficiently large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* $(0 < p^* < 1)$, t_q^0 and c, the smallest positive integer, n required to assert that $t_q > t_q^0$ must satisfy:

$$\sum_{i=0}^{c} \binom{n}{i} \, p_0^i \, (1 - p_0)^{n-i} \le 1 - p^*, \qquad (5)$$

where $p = F(t;\delta_0)$ is the probability of a failure during the time t given a specified 100qth percentile of lifetime t_q^0 and depends only on $\delta_0 = t/t_q^0$, since $\partial F(t;\delta)/\partial\delta > 0$, $F(t;\delta)$ is a non-decreasing function of δ. Accordingly, we have:

$$F(t, \delta) < F(t, \delta_0) \Leftrightarrow \delta \le \delta_0,$$

Or equivalently,

$$F(t, \delta) \le F(t, \delta_0) \Leftrightarrow t_q \ge t_q^0.$$

The smallest sample size n satisfying the inequality (5) can be obtained for any given q, t/t_q^0, p^*. Whereas, the smallest sample size n calculation in Rosaiah and Kantam [20] only needs input values for t/σ_0 and p^*. Hence, the proposed process to find the smallest sample size in this case is the same as the procedure provided by Rosaiah and Kantam [20] for the inverse Rayleigh model except in place of t/σ_0 replace by t/t_q^0 at q. To save space, only the results of small sample sizes for q=0.1, t/t_q^0=0.7, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5; p^*=0.75, 0.90, 0.95, 0.99; c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are reported in Table 1.

Table 1. Minimum sample sizes necessary to assert the 10th percentile to exceed a given values, $t_{0.1}^{0}$, with probability *p and the corresponding acceptance number, *c*, for the inverse Rayleigh distribution using the binomial approximation.**

p^*	c	$t / t_{0.1}^{0}$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	10	5	4	2	1	1	1	1
0.75	1	20	9	7	4	3	3	2	2
0.75	2	30	13	10	5	4	4	4	3
0.75	3	39	17	13	7	6	5	5	5
0.75	4	48	21	16	9	7	6	6	6
0.75	5	56	25	19	11	9	8	7	7
0.75	6	65	29	22	12	10	9	8	8
0.75	7	74	32	25	14	11	10	9	9
0.75	8	82	36	28	16	13	11	11	10
0.75	9	91	40	31	17	14	13	12	11
0.75	10	99	44	34	19	15	14	13	12
0.90	0	17	7	6	3	2	2	2	1
0.90	1	29	12	10	5	4	3	3	3
0.90	2	40	17	13	7	5	5	4	4
0.90	3	50	22	17	9	7	6	5	5
0.90	4	60	26	20	11	8	7	7	6
0.90	5	70	30	23	12	10	9	8	7
0.90	6	79	34	27	14	11	10	9	9
0.90	7	89	38	30	16	13	11	10	10
0.90	8	98	42	33	18	14	12	12	11
0.90	9	107	47	36	20	15	14	13	12
0.90	10	116	51	39	21	17	15	14	13
0.95	0	22	9	7	3	2	2	2	2
0.95	1	35	15	11	6	4	4	3	3
0.95	2	47	20	15	8	6	5	5	4
0.95	3	58	25	19	10	7	6	6	6
0.95	4	68	29	23	12	9	8	7	7
0.95	5	79	34	26	14	10	9	8	8
0.95	6	89	38	30	15	12	10	10	9
0.95	7	99	42	33	17	13	12	11	10
0.95	8	108	47	36	19	15	13	12	11
0.95	9	118	51	40	21	16	14	13	13
0.95	10	127	55	43	23	18	16	15	14
0.99	0	34	14	11	5	4	3	3	2
0.99	1	49	20	16	8	5	5	4	4
0.99	2	62	26	20	10	7	6	5	5
0.99	3	74	31	24	12	9	8	7	6
0.99	4	86	36	28	14	11	9	8	8
0.99	5	97	41	32	16	12	10	9	9
0.99	6	108	46	35	18	14	12	11	10
0.99	7	119	51	39	20	15	13	12	11
0.99	8	130	55	43	22	17	15	13	13
0.99	9	140	60	46	24	18	16	15	14
0.99	10	150	64	50	26	20	17	16	15

Table 2. Minimum sample sizes necessary to assert the 10th percentile to exceed a given values, $t^0_{0.1}$, with probability *p and the corresponding acceptance number, *c*, for the inverse Rayleigh distribution using the Poisson approximation.**

p^*	c	$t/t^0_{0.1}$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	11	5	4	3	2	2	2	2
0.75	1	18	8	7	4	3	3	3	3
0.75	2	30	13	11	6	5	5	5	5
0.75	3	39	18	14	8	7	6	6	6
0.75	4	49	22	18	10	9	8	7	7
0.75	5	58	26	21	12	10	9	9	9
0.75	6	66	30	24	14	11	11	10	10
0.75	7	75	34	27	16	13	12	11	11
0.75	8	84	38	30	17	14	13	13	12
0.75	9	92	41	33	19	16	14	14	13
0.75	10	101	45	36	21	17	16	15	15
0.90	0	18	8	7	4	3	3	3	3
0.90	1	28	13	10	6	5	5	5	4
0.90	2	41	19	15	9	7	7	6	6
0.90	3	52	23	19	11	9	8	8	8
0.90	4	62	28	22	13	11	10	9	9
0.90	5	72	32	26	15	12	11	11	11
0.90	6	82	37	29	17	14	13	12	12
0.90	7	91	41	32	19	16	14	14	13
0.90	8	101	45	36	21	17	16	15	15
0.90	9	110	49	39	23	19	17	16	16
0.90	10	119	53	42	25	20	19	18	17
0.95	0	24	11	9	5	4	4	4	4
0.95	1	35	16	13	8	6	6	6	5
0.95	2	49	22	17	10	9	8	7	7
0.95	3	60	27	22	13	10	10	9	9
0.95	4	71	32	25	15	12	11	11	10
0.95	5	81	37	29	17	14	13	12	12
0.95	6	92	41	33	19	16	14	14	13
0.95	7	102	46	36	21	17	16	15	15
0.95	8	112	50	40	23	19	17	17	16
0.95	9	121	54	43	25	21	19	18	18
0.95	10	131	59	47	27	22	20	19	19
0.99	0	36	16	13	8	6	6	6	5
0.99	1	50	23	18	11	9	8	8	8
0.99	2	65	29	23	14	11	10	10	10
0.99	3	78	35	28	16	13	12	12	11
0.99	4	90	40	32	19	15	14	13	13
0.99	5	101	46	36	21	17	16	15	15
0.99	6	113	51	40	23	19	18	17	16
0.99	7	124	55	44	25	21	19	18	18
0.99	8	134	60	48	28	23	21	20	19
0.99	9	145	65	52	30	25	23	21	21
0.99	10	156	70	55	32	26	24	23	22

If $p = F(t;\delta_0)$ is small and n is large the binomial probability may be approximated by Poisson probability with parameter $\lambda = np$ so that the left side of (5) can be written as:

$$\sum_{i=0}^{c} \frac{\lambda^i}{i\,!} e^{-\lambda} \leq 1 - p^*, \qquad (6)$$

where $\lambda = n\,F(t;\delta_0)$.

2.2 Operating Characteristic of the Sampling Plan $(n, c, t/t_q^0)$

The operating characteristic (OC) function of the sampling plan $(n, c, t/t_q^0)$ is the probability of accepting a lot. It is given as:

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i}, \qquad (7)$$

where $p = F(t;\delta)$. It should be noticed that $F(t;\delta)$ can be represented as a function of $\delta = t/t_q$. Therefore, $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$ where $d_q = t_q/t_q^0$. Using Eq. (7), the OC values and OC curves can be obtained for any sampling plan $(n, c, t/t_q^0)$. To save space, we present Table 3 to show the OC values for the sampling plan $(n, c = 4, t/t_{0.1}^0)$. Figure1 shows the OC curves for the sampling plan $(n, c, t/t_{0.1}^0)$ with p^*=0.75 for $\delta_0 = 1$, where c = 0,1,2,3,4,5,6,7,8,9,10.

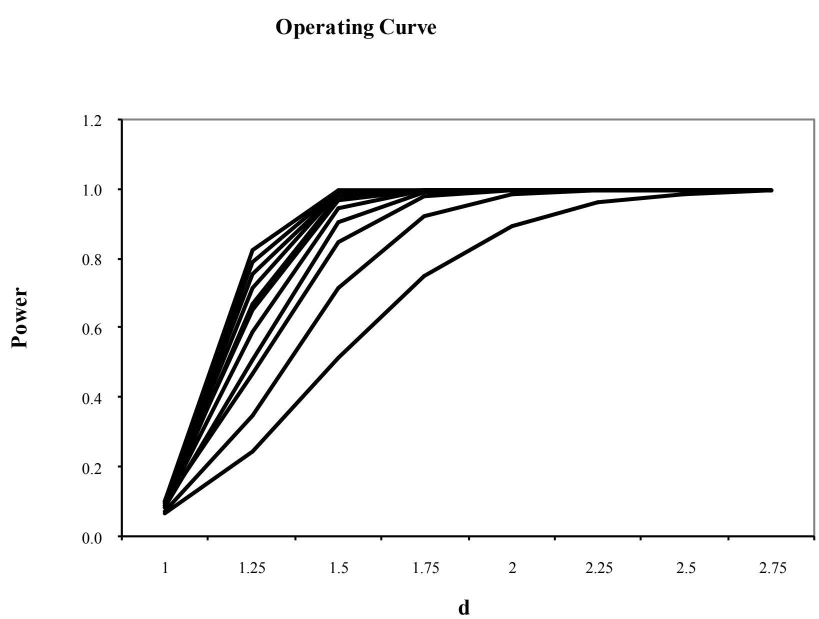

Figure 1. OC curves for c = 0,1,2,3,4,5,6,7,8,9,10, respectively under p^*=0.90, $\delta_0 = 1$ based on the 10th percentile, $d = d_{0.1}$, of inverse Rayleigh distribution.

Table 3. Operating characteristic values of the sampling plan $(n, c = 5, t/t_{0.1}^0)$ for a given p^* under inverse Rayleigh distribution.

			$t_{0.1}/t_{0.1}^0$							
p^*	n	$t/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
0.75	56	0.70	0.2483	0.9726	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.75	25	0.90	0.2214	0.8554	0.9963	1.0000	1.0000	1.0000	1.0000	1.0000
0.75	19	1.00	0.2432	0.8083	0.9890	0.9998	1.0000	1.0000	1.0000	1.0000
0.75	11	1.50	0.1640	0.5018	0.8194	0.9614	0.9950	0.9996	1.0000	1.0000
0.75	9	2.00	0.1162	0.3254	0.5919	0.8099	0.9323	0.9814	0.9961	0.9993
0.75	8	2.50	0.1016	0.2515	0.4541	0.6577	0.8167	0.9163	0.9673	0.9891
0.75	7	3.00	0.1626	0.3097	0.4807	0.6455	0.7809	0.8775	0.9379	0.9715
0.75	7	3.50	0.0990	0.2006	0.3334	0.4807	0.6234	0.7461	0.8409	0.9073
0.90	70	0.70	0.0941	0.9313	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.90	30	0.90	0.0928	0.7360	0.9906	0.9999	1.0000	1.0000	1.0000	1.0000
0.90	23	1.00	0.0973	0.6521	0.9715	0.9995	1.0000	1.0000	1.0000	1.0000
0.90	12	1.50	0.0955	0.3890	0.7463	0.9392	0.9915	0.9993	1.0000	1.0000
0.90	10	2.00	0.0489	0.1932	0.4442	0.7055	0.8824	0.9646	0.9919	0.9986
0.90	9	2.50	0.0323	0.1162	0.2757	0.4856	0.6895	0.8420	0.9323	0.9755
0.90	8	3.00	0.0434	0.1215	0.2515	0.4188	0.5931	0.7445	0.8563	0.9276
0.90	7	3.50	0.0990	0.2006	0.3334	0.4807	0.6234	0.7461	0.8409	0.9073
0.95	79	0.70	0.0464	0.8924	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
0.95	34	0.90	0.0425	0.6277	0.9828	0.9999	1.0000	1.0000	1.0000	1.0000
0.95	26	1.00	0.0446	0.5286	0.9504	0.9989	1.0000	1.0000	1.0000	1.0000
0.95	14	1.50	0.0288	0.2134	0.5858	0.8768	0.9796	0.9981	0.9999	1.0000
0.95	10	2.00	0.0489	0.1932	0.4442	0.7055	0.8824	0.9646	0.9919	0.9986
0.95	9	2.50	0.0323	0.1162	0.2757	0.4856	0.6895	0.8420	0.9323	0.9755
0.95	8	3.00	0.0434	0.1215	0.2515	0.4188	0.5931	0.7445	0.8563	0.9276
0.95	8	3.50	0.0200	0.0605	0.1370	0.2515	0.3936	0.5445	0.6838	0.7976
0.99	97	0.70	0.0098	0.7887	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	41	0.90	0.0094	0.4404	0.9601	0.9997	1.0000	1.0000	1.0000	1.0000
0.99	32	1.00	0.0079	0.3115	0.8856	0.9967	1.0000	1.0000	1.0000	1.0000
0.99	16	1.50	0.0077	0.1060	0.4300	0.7931	0.9598	0.9957	0.9997	1.0000
0.99	12	2.00	0.0071	0.0570	0.2169	0.4843	0.7463	0.9085	0.9757	0.9952
0.99	10	2.50	0.0093	0.0489	0.1543	0.3356	0.5543	0.7493	0.8824	0.9540
0.99	9	3.00	0.0100	0.0415	0.1162	0.2443	0.4133	0.5919	0.7474	0.8612
0.99	9	3.50	0.0034	0.0157	0.0491	0.1162	0.2230	0.3625	0.5165	0.6628

2.3 Producer's Risk

The producer's risk is defined as the probability of rejecting the lot when $t_q > t_q^0$. For a given value of the producer's risk, say α, we are interested in knowing the value of d_q to ensure the producer's risk is less than or equal to α if a sampling plan $(n, c, t/t_q^0)$ is developed at a specified confidence level p^*. Thus, one needs to find the smallest value d_q according to Eq. (7) as:

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1-\alpha, \qquad (8)$$

where $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$, $d_q = t_q / t_q^0$.

Table 4. - Minimum ratio of true $d_{0.1}$ for the acceptability of a lot for the inverse Rayleigh distribution and producer's risk of 0.05.

p^*	c	$t/t_{0.1}^0$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	2.5031	2.7631	2.8761	3.4621	4.3273	5.1953	6.0613	6.9247
0.75	1	1.6095	1.7126	2.291	2.8337	3.5398	3.6832	4.2918	4.9188
0.75	2	1.4859	1.5625	1.9391	2.3624	2.9533	3.5398	3.4990	4.0080
0.75	3	1.4164	1.4786	1.8335	2.2852	2.5913	3.1153	3.6245	4.1545
0.75	4	1.3695	1.4231	1.7658	2.0764	2.3381	2.8074	3.2723	3.7439
0.75	5	1.3373	1.3841	1.7159	2.0670	2.3935	2.5767	3.0021	3.4329
0.75	6	1.3141	1.3552	1.6124	1.9350	2.2346	2.3935	2.7988	3.1918
0.75	7	1.2844	1.3314	1.5926	1.8262	2.1053	2.2512	2.6212	3.0021
0.75	8	1.2700	1.3122	1.5733	1.8447	1.9944	2.3935	2.4783	2.8337
0.75	9	1.2577	1.2972	1.5081	1.7658	2.0670	2.2795	2.3563	2.6991
0.75	10	1.2473	1.2825	1.5006	1.6935	1.9814	2.1810	2.2512	2.5767
0.90	0	2.5231	2.6541	3.0295	3.8361	4.7952	5.7544	6.0613	6.9247
0.90	1	1.6841	1.8188	2.4062	3.0525	3.5398	4.2517	4.9727	5.6883
0.90	2	1.5598	1.6504	2.1501	2.5840	3.2258	3.5398	4.1356	4.7393
0.90	3	1.4908	1.5733	2.0032	2.4450	2.8604	3.1153	3.6245	4.1545
0.90	4	1.4343	1.5056	1.9066	2.2292	2.5988	3.1153	3.2723	3.7439
0.90	5	1.3947	1.4573	1.7797	2.1863	2.5840	2.8785	3.0021	3.4329
0.90	6	1.3633	1.4343	1.7322	2.0530	2.4190	2.6831	3.1368	3.5817
0.90	7	1.3392	1.4033	1.6966	2.0392	2.2795	2.5265	2.9438	3.3693
0.90	8	1.3198	1.3778	1.6686	1.9433	2.1654	2.5988	2.7902	3.1918
0.90	9	1.3103	1.3572	1.6415	1.8598	2.2075	2.4851	2.6596	3.0423
0.90	10	1.2972	1.3392	1.5843	1.8636	2.1151	2.3810	2.5478	2.9061
0.95	0	2.5631	2.8521	3.0295	3.8361	4.7952	5.7544	6.7150	7.6728
0.95	1	1.7388	1.8447	2.4988	3.0525	3.8226	4.2517	4.9727	5.6883
0.95	2	1.6038	1.6935	2.2237	2.7397	3.2258	3.8715	4.1356	4.7393
0.95	3	1.5258	1.6095	2.0670	2.4450	2.8604	3.4329	4.0080	4.5725
0.95	4	1.4667	1.5545	1.9643	2.3563	2.7902	3.1153	3.6390	4.1545
0.95	5	1.4320	1.5006	1.8868	2.1863	2.5840	2.8785	3.3568	3.8388
0.95	6	1.3990	1.4715	1.7832	2.1501	2.4190	2.9061	3.1368	3.5817
0.95	7	1.3716	1.4388	1.7422	2.0392	2.4254	2.7397	2.9438	3.3693
0.95	8	1.3552	1.4098	1.7094	2.0255	2.3084	2.5988	2.7902	3.1918
0.95	9	1.3373	1.3968	1.6810	1.9391	2.2075	2.4851	2.8969	3.3080
0.95	10	1.3217	1.3778	1.6565	1.9391	2.2292	2.5407	2.7732	3.1696
0.99	0	2.5431	2.9531	3.2125	4.1782	5.0482	6.0613	6.7150	7.6728
0.99	1	1.8044	1.9474	2.6288	3.2144	4.0258	4.5956	5.3533	6.1087
0.99	2	1.6686	1.7797	2.3441	2.8694	3.4329	3.8715	4.5269	5.1706
0.99	3	1.5843	1.6841	2.1758	2.6752	3.2144	3.6684	4.0080	4.5725
0.99	4	1.5284	1.6181	2.0576	2.5407	2.9438	3.3445	3.9047	4.4603
0.99	5	1.4859	1.5706	1.9728	2.3747	2.7315	3.1046	3.6245	4.1356
0.99	6	1.4550	1.5258	1.9066	2.3143	2.6911	3.0836	3.3818	3.8715
0.99	7	1.4298	1.4981	1.8560	2.1968	2.5478	2.9155	3.1918	3.6536
0.99	8	1.4033	1.4762	1.8116	2.1654	2.5336	2.7732	3.2373	3.6982
0.99	9	1.3862	1.4480	1.7762	2.0764	2.4254	2.7902	3.0941	3.5261
0.99	10	1.3674	1.4343	1.7455	2.0623	2.3321	2.6752	2.9630	3.3944

To save space, based on sampling plans $(n, c, t/t_q^0)$ established in Tables 1 the minimum ratios of $d_{0.1}$ for the acceptability of a lot at the producer's risk of α =0.05 are presented in Table 4.

3. Illustrative Examples

In this section, two examples with real data sets are given to illustrate the proposed acceptance sampling plans. The first data set is of the data given arisen in tests on endurance of deep groove ball bearings (Lawless [11], p.228). The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04 and 173.40. The second data set is obtained from Proschan [15] and represents times between successive failures of air conditioning (AC) equipment in a Boeing 720 airplane and they are as follows: 12, 21, 26, 27, 29, 29, 48, 57, 59, 70, 74, 153, 326, 386 and 502.

As the confidence level is assured by this acceptance sampling plan only if the lifetimes are from the inverse Rayleigh distribution. Then, we should check if it is reasonable to admit that the given sample comes from the Inverse Rayleigh distribution by the goodness of fit test and model selection criteria. The first data set was used by Sultan [23] to demonstrate the goodness of fit for generalized exponential distribution and Gupta and Kundu [8] fitted for extended exponential distribution. However, the acceptance sampling plans under the truncated life test based on the Inverse Rayleigh distribution for percentiles has not yet been developed. We fit the inverse Rayleigh distribution to the two data sets separately. We used the Kolmogorov-Smirnov (K-S) tests for each data set to the fit the inverse Rayleigh model. It is observed that for Data Sets I and II, the K-S distances are 0.12091 and 0.21378 with the corresponding p values are 0.85028 and 0.43879 respectively. For data sets I and II, the chi-square values are 0.3052 and 2.6383 respectively. Therefore, it is clear that inverse Rayleigh model fits quite well to both the data sets.

3.1 Example 1

Assume that the lifetime distribution is inverse Rayleigh distribution and that the experimenter is interested to establish the true unknown 10th percentile lifetime for the ball bearings to be at least 30 million revolutions with confidence p^*=0.90 and the life test would be ended at 30 million revolutions, which should have led to the ratio $t/t_{0.1}^0$ = 1.0. Thus, for an acceptance number c =5 and the confidence level p^*=0.90, the required sample size n found from Table 1 should be at least 23. Therefore, in this case, the acceptance sampling plan from truncated life tests for the inverse Rayleigh distribution 10th percentile should be $(n, c, t/t_q^0)$= (23, 5, 1.0). Based on the ball bearings data, the experimenter must have decided whether to accept or reject the lot. The lot should be accepted only if the number of items of which lifetimes were less than or equal to the scheduled test lifetime, 30 million revolutions, was at most 5 among the first 23 observations. Since there were 2 items with a failure time less than or equal to 30 million revolutions in the given sample of n =23 observations, the experimenter would accept the lot, assuming the 10th percentile lifetime $t_{0.1}$ of at least 30 million revolutions with a confidence level of p^*=0.90. The

OC values for the acceptance sampling plan $(n,c,t/t_q^0)$= (23,5,1.0) and confidence level p^*=0.90 under inverse Rayleigh distribution from Table 3 is as follows:

$t_{0.1}/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
OC	0.0973	0.6521	0.9715	0.9995	1.0000	1.0000	1.0000	1.0000

This shows that if the true 10th percentile is equal to the required 10th percentile ($t_{0.1}/t_{0.1}^0$= 1.00) the producer's risk is approximately 0.9027 (=1- 0.0973). The producer's risk is almost equal to zero when the true 10th percentile is greater than or equal to 2.00 times the specified 10th percentile.

From Table 4, the experimenter could get the values of $d_{0.1}$ for different choices of c and $t/t_{0.1}^0$ in order to assert that the producer's risk was less than 0.05. In this example, the value of $d_{0.1}$ should be 1.7797 for c = 5, $t/t_{0.1}^0$=1.0 and p^*=0.90. This means the product can have a 10th percentile life of 1.7797 times the required 10th percentile lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95.

Alternatively, assume that products have an inverse Rayleigh distribution and consumers wish to reject a bad lot with probability of p^*=0.75. What should the true 10th percentile life of products be so that the producer's risk is 0.05 if the acceptance sampling plan is based on an acceptance number c =5 and $t/t_{0.1}^0$=0.7? From Table 4, we can find that the entry for p^*=0.75, c = 5, and $t/t_{0.1}^0$=0.7 is $d_{0.1}$= 1.3373. Thus, the manufacturer's product should have a 10th percentile life at least 1.3373 times the specified 10th percentile life in order for the products to be accepted with probability 0.75 under the above acceptance sampling plan. Table 1 indicates that the number of products required to be tested is n = 56 so that the sampling plan is $(n,c,t/t_{0.1}^0)$= (56, 5, 0.7).

3.2 Example 2

Suppose an experimenter would like to establish the true unknown 10th percentile lifetime for the data set regarding the failure of air conditioning (AC) equipment in a Boeing 720 airplane mentioned above to be at least 20 and the life test would be ended at 20, which should have led to the ratio $t/t_{0.1}^0$= 1.00. The goodness of fit test for these 15 observations were verified and showed that inverse Rayleigh model as a reasonable goodness of fit for these 15 observations. Thus, with c = 2 and p^*=0.95, the experimenter should find from Table 1 the sample size n must be at least 15 and the sampling plan to be $(n,c,t/t_{0.1}^0)$= (15, 2, 1.00). Since there is a one item with a failure time less than 20 in the given sample of n = 15 observations, the experimenter would accept the lot, assuming the 10th percentile lifetime $t_{0.1}$ of at least 20 with a confidence level of p^*=0.95.

4. Future Work

Construction of these sample plans with reference to population percentiles is in progress by the authors in other ramifications also such as double, sequential, two-stage and interval censored group samples.

5. Discussion and Conclusions

The sampling plans based on the inverse Rayleigh population mean developed by Rosaiah and Kantam [20] to the inverse Rayleigh models. It shows that the minimum sample sizes are smaller than those reported in Tables 1 and 2 of this article for the 10th percentile for both binomial and Poisson approximation. Here, $\delta_0 = t/t_{0.1}^0$ for the sampling plans based on 10th percentile is replaced by $\delta_0 = t/\mu_0$ with μ_0 as a specific population mean for the acceptance plans based on the inverse Rayleigh population mean. Therefore, the acceptance sampling plans based on the inverse Rayleigh population mean could have less chance to report a failure than the acceptance sampling plans based on 10th percentile. The acceptance sampling plans based on population mean could accept the lot of bad quality of the 10th percentiles. The minimum sample sizes are reported in Table 1 of this article for the 10th percentiles are compared with the minimum sample sizes are reported in Table 1 of Lio *et al*. [12]. It shows that the minimum sample sizes using inverse Rayleigh population are smaller than those reported in Tables 1 of Lio *et al*. [12] for Birnbaum-Saunders population for the 10th percentile when $\delta_0 \leq 1.0$ whereas, the minimum sample sizes using inverse Rayleigh population are larger than those reported in Tables 1 of Lio *et al*. [12] for Birnbaum-Saunders population for the 10th percentile when $\delta_0 > 1.0$.

This article has derived the acceptance sampling plans based on the inverse Rayleigh percentiles when the life test is truncated at a pre-fixed time. The procedure is provided to construct the proposed sampling plans for the percentiles of the inverse Rayleigh distribution. To ensure that the life quality of products exceeds a specified one in terms of the life percentile, the acceptance sampling plans based on percentiles should be used. Some useful tables are provided and applied to establish acceptance sampling plans for two examples.

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