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RELIABILITY EVALUATION OF BRIDGE SYSTEM WITH FUZZY RELIABILITY OF COMPONENTS USING INTERVAL NONLINEAR PROGRAMMING

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Abstract: *Practically reliability of component is imprecise in nature. We have considered reliability components are taken fuzzy in nature, so system reliability is also in fuzzy nature. We apply Zadeh's extension principal and interval arithmetic operation to evaluate reliability of the system assembled with fuzzy reliability of components. We approximate the system reliability to a fuzzy number and evaluate the corresponding maximum divergence. Here, we demonstrate a bridge network whose components reliability are taken as triangular fuzzy number. Evaluating reliability of the system belongs to an interval. We have considered cost constrained goal as an interval number, interval nonlinear programming is used to find out the system reliability. Numerical examples are given to illustrate the approach to evaluate the system reliability.*

Keywords: *Reliability, fuzzy number, interval number, bridge network, interval nonlinear programming.*

1. Introduction

Formulation of reliability optimization problems assume that coefficients are known as fixed quantities and treated as conventional optimization problem. Very often, the data used to evaluate the reliability of the system and components may come from different sources. Some data are of objective measure and based on well established statistical models, while others may possibly be supplied by experts. Therefore, this assumption of precise probability distributions of

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the component time to failure may be unreasonable, or may be violated. Kozine and Filimonov [16] presented imprecise system reliability assessment based on coherent imprecise probabilities. Coolen [8] discussed applications of imprecise probabilities in reliability.

Fuzzy set [25] is very appropriate to describe the reliability. To solve the problem of incomplete available information, fuzzy reliability theory has been proposed for better significance. Cai *et al*. [2] presented two fundamental assumptions in conventional reliability theory. These two assumptions are binary state assumption and probability assumption. The estimation of precise values of probabilities is very difficult in many systems due to the uncertainties and inaccuracy of data. Cheng and Mon [5] used interval arithmetic operations of fuzzy numbers to evaluate fuzzy system reliability. They presented a method for fuzzy system reliability analysis by interval of confidence. In [19] it was supposed that components were with different membership functions and interval arithmetic and α-cuts were used to evaluate fuzzy system reliability. Chen [6] presented a method to analyze fuzzy system reliability using simplified fuzzy arithmetic operations of fuzzy numbers, where the reliability of each system component is represented by a triangular fuzzy number. Hong et al. [11] analyzed fuzzy system reliability by the use of t-norm based convolution of fuzzy arithmetic operation, where the reliability of each component is represented by L-R type fuzzy numbers. There is a good book dealing with fuzzy reliability by Cai [3].

The bridge system structure has widely used in a system design in addition to series system structure, and parallel system structure. Most methods are based on conventional reliability theory ([4],[9],[17]). Chen [7] applied fuzzy reliability theory to analyze the bridge system. Sun et al. [22] proposed convexification method for solving a class of global optimization problems with application to reliability optimization where two forms of the bridge system optimization model have considered; one in reliability maximization with cost constraints and the other is cost minimization with system reliability constraint goal. Pan et al. [20], Misra and Sharma [18], and Tzafertas [24] considered bridge network problem as a complex system. Gopal et al. [10] presented redundancy optimization for bridge system as an example of complex network which is broken into several simpler and non-interacting smaller problem of optimization for series network.

In fuzzy programming approach, the coefficients are viewed as fuzzy sets and their membership functions are assumed to be known. However, in many cases of application of real world problems, it is not so easy for decision makers to specify either probability distributions or membership functions. On the contrary, such uncertainty can easily be represented by an interval. This is the motivation to develop interval arithmetic and interval programming [13]. The interval programming technique is developed to deal with the mathematical programming problem with interval coefficients. Ishibuchi and Tanaka [12] have proposed a solution procedure to interval programming problem, the essential of which is to transform the problem into an equivalent bicriteria programming problem. Tong [23] investigated the problems in which the coefficients of the objective function and the constraints are all interval numbers. Ashchepkov [1] proposed reductions of a general nonlinear programming problem with interval functions to the deterministic nonlinear programming problem. Jiang et al. [14] presented a method to solve the nonlinear interval number programming problem with uncertain coefficients both in nonlinear objective function and nonlinear constraints.

A tool for performing arithmetic and logical operations in the fuzzy environment is the min-operator originally proposed by L.A. Zadeh and known as Zadeh's Extension principle which is congruent with results of engineering experiments.

In this paper, approximation of fuzzy number and Zadeh's extension principal is used to evaluate reliability of bridge system, whose components reliability are considered as triangular fuzzy number (TFN). We have examined the optimal design problem of system reliability with uncertainty. The cost constraint goal is considered as an interval number. Then interval nonlinear programming is used to find out the system reliability. We formulate an interval optimization problem of system reliability as a non-linear function in interval which is formulated first as interval nonlinear programming problem. Then we have transformed the interval programming problem into equivalent problems from which solutions can be derived more simply and efficiently.

2. Model formulation

Notation:
A bridge system reliability model is developed under the following notations:

r_i	reliability of component of reliability model for i[th] components,
$\tilde{r}_i$	fuzzy reliability of component of reliability model for i[th] components,
$\left[r_{i1}(\alpha), r_{i2}(\alpha)\right]$	α-cut of fuzzy reliability of component for i[th] components,
$\left[C^L, C^U\right]$	available system cost interval of the reliability model,
$R_s(r_1, r_2, r_3, r_4, r_5)$	system reliability function of the reliability model,
$\tilde{R}_s(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4, \tilde{r}_5)$	fuzzy system reliability function of the reliability model,
$C_s(r_1, r_2, r_3, r_4, r_5)$	system cost function of the reliability model,
$\tilde{C}_s(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4, \tilde{r}_5)$	fuzzy system cost function of the reliability model,
$\left[R_s^L(\alpha), R_s^U(\alpha)\right]$	α-cut of fuzzy system reliability,
$\left[R_{SA}^L(\alpha), R_{SA}^U(\alpha)\right]$	α-cut of approximated triangular fuzzy system reliability,

2.1 Crisp model of Bridge system

The bridge system is being considered as a system of five components for finding out the system reliability. The algebraic expression and pictorial form (figure-1) of the bridge given as follows:

$$R_s(r_1, r_2, r_3, r_4, r_5) = r_5(r_1 + r_2 - r_1 r_2)(r_3 + r_4 - r_3 r_4) + (1 - r_5)(r_1 r_3 + r_2 r_4 - r_1 r_2 r_3 r_4) \quad (1)$$

such that $0 < r_i \le 1$ for i=1,2,3,4,5.

The cost constraints for bridge system is given by:

$$C_s(r_1, r_2, r_3, r_4, r_5) = \sum_{i=1}^{5} a_i \exp\left(\frac{b_i}{1 - r_i}\right) \quad (2)$$

where a_i *and* b_i are shape parameters.

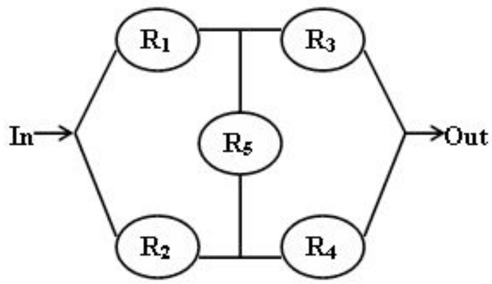

Figure 1. Diagram of five unit bridge network.

2.2 Bridge system in fuzzy environment

In real life, the reliability of components is more realistic to consider them as fuzzy. The system reliability (1) of bridge system are expressed as fuzzy system reliability function of fuzzy reliability of components as follows:

$$\begin{aligned}\tilde{R}_s\left(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3,\tilde{r}_4,\tilde{r}_5\right) = {} & 2\tilde{r}_1\tilde{r}_2\tilde{r}_3\tilde{r}_4\tilde{r}_5 - \tilde{r}_1\tilde{r}_3\tilde{r}_4\tilde{r}_5 - \tilde{r}_2\tilde{r}_3\tilde{r}_4\tilde{r}_5 - \tilde{r}_1\tilde{r}_2\tilde{r}_3\tilde{r}_5 - \tilde{r}_1\tilde{r}_2\tilde{r}_4\tilde{r}_5 - \tilde{r}_1\tilde{r}_2\tilde{r}_3\tilde{r}_4 \\ & + \tilde{r}_1\tilde{r}_4\tilde{r}_5 + \tilde{r}_2\tilde{r}_3\tilde{r}_5 + \tilde{r}_1\tilde{r}_3 + \tilde{r}_2\tilde{r}_4\end{aligned} \tag{3}$$

such that $0 < \tilde{r}_i \leq 1$ for i=1,2,3,4,5.

Let a constraint goal is imposed by the decision-maker. It is not always possible for the decision-maker to estimate the cost of the system in exact way, but it is easy to estimate the cost of the system in a range of his capacity or relevance. Thus the constraint goal is more meaningful by consider an interval $\left[C^L, C^U\right]$. The cost constraints in fuzzy environment for the bridge system is as follows:

$$C_s\left(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3,\tilde{r}_4,\tilde{r}_5\right) = \sum_{i=1}^{5} a_i \exp\left(\frac{b_i}{1-\tilde{r}_i}\right) \subseteq \left[C^L, C^U\right] \tag{4}$$

3. Prerequisite Mathematics

Definition 3.1. Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) : x \in X\right\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(x)$ is the stronger the grade of membership in $\tilde{A}$.

Definition 3.2. α-Level Set or α-cut of a Fuzzy Set: The α-level set or interval of confidence at level α or α-cut of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in $\tilde{A}$ greater than or equal to α i.e. $A_\alpha = \left\{x : \mu_{\tilde{A}}\left(x\right) \geq \alpha, x \in X, \alpha \in [0,1]\right\}$.

Definition 3.3. Triangular Fuzzy Number: Let F(R) be a set of all triangular fuzzy numbers in real line R. A triangular fuzzy number $\tilde{A} \in F\left(R\right)$ is a fuzzy number with the membership function $\mu_{\tilde{A}} : R \rightarrow \left[0,1\right]$ (Figure 2) parameterized by a triplet (a_1, a_2, a_3)$_{TFN}$. Where a_1 and a_3 denote the lower and upper limits of support of a fuzzy $\tilde{A}$:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a_2 - x}{a_2 - a_1} \; for \; a_1 \leq x \leq a_2 \\ 1 - \dfrac{x - a_2}{a_3 - a_2} \; for \; a_2 \leq x \leq a_3 \\ 0 \quad otherwise \end{cases}$$

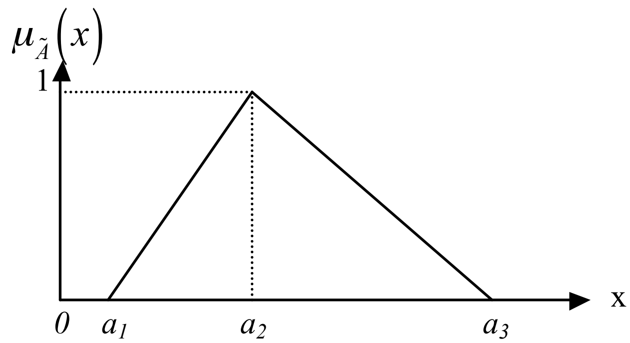

Figure 2. Triangular Fuzzy Number.

Definition 3.4. Zadeh's Extension Principle: Let $f : \underset{i=1}{\overset{n}{\times}} \rightarrow Y$ be a mapping. Then the extension principle allows us to define the fuzzy set $\tilde{B}$ in Y induced by the fuzzy set $\underset{i=1}{\overset{n}{\times}} \tilde{A}_i$ in $\underset{i=1}{\overset{n}{\times}} X_i$ through f as follows:

$$\tilde{B} = \left\{\left(y, \mu_{\tilde{B}}\left(y\right)\right) : y = f\left(x_1, x_2, ..., x_n\right), \left(x_1, x_2, ..., x_n\right) \in X_1 \times X_2 \times .... \times X_n\right\},$$

with $\mu_B\left(y\right) = \begin{cases} \underset{\substack{(x_1, x_2, ..., x_n) \in X_1 \times X_2 \times ... \times X_n \\ y = f(x_1, x_2, ..., x_n)}}{Sup} \mu_{A_1 \times A_2 \times ... \times A_n}\left(x_1, x_2, ..., x_n\right) & f^{-1}\left(y\right) \neq \phi \\ 0 & f^{-1}\left(y\right) = \phi \end{cases}$.

Where $f^{-1}(y)$ is the inverse image of y. If all of the fuzzy sets $\tilde{A}_1,\tilde{A}_2,...,\tilde{A}_n$ are fuzzy numbers then the use of Zadeh's extension principle is significant by the following theorem

Theorem 3.1. Let $y = f(x_1, x_2, ..., x_n)$ be a real-valued ordinary continuous function with n variables $x_1, x_2, ..., x_n$. Also let $\tilde{A}_1,\tilde{A}_2,...,\tilde{A}_n$ be fuzzy numbers with their corresponding $\alpha-$level sets denoted by $\left[x_1^L(\alpha), x_1^U(\alpha)\right], \left[x_2^L(\alpha), x_2^U(\alpha)\right], ..., \left[x_n^L(\alpha), x_n^U(\alpha)\right]$. Then the $\alpha-$level set of the image of $\tilde{A}_1,\tilde{A}_2,...,\tilde{A}_n$ through f denoted by $f(x_1, x_2, ..., x_n)$ can be represented by $\left[\tilde{B}\right]_\alpha = \left\{ f(x_1, x_2, ..., x_n) \in R^1 : x_i \in \left[x_{i\alpha}^L, x_{i\alpha}^U\right], i = 1, 2, ..., n \right\}$. Proof: (See [21])

3.1 *Solution procedure of interval nonlinear programming*

Let the objective function $f(x)$ is given in the form of interval where both the lower interval $f^L(x)$ and upper interval $f^U(x)$ are nonlinear functions. The constraints are also given in the form of interval of nonlinear functions with interval constraints goal. Then the standard form is:

$$\text{Max } f(x) = \left[f^L(x), f^U(x)\right] \tag{5}$$

Subject to $g_i(x) = \left[g_i^L(x), g_i^U(x)\right] \subseteq \left[G_i^L, G_i^U\right]$ for i=1,2,…,m

Where x is an n-dimensional decision vector, $f(x)$ and $g_i(x)$ are objective function and the ith constraints respectively. According to the operation of interval, the constraint inequality of problem (5) can be transformed into $g_i^L(x) \geq G_i^L$ and $g_i^U(x) \leq G_i^U$.

Theorem 3.2. Suppose $\left[g_i^L(x), g_i^U(x)\right] \subseteq \left[c_i^L, c_i^U\right]$ then $c_i^U \geq g_i^U(x)$, $c_i^L \leq g_i^L(x)$ are maximum and minimum value range inequality respectively for this constraint condition [23].

According to the theorem 3.2 for each constraint condition $\left[g_i^L(x), g_i^U(x)\right] \subseteq \left[G_i^L, G_i^U\right]$ of the interval nonlinear programming problem (5), there is maximum value range inequality $G_i^U \geq g_i^U(x)$ and minimum value range inequality $G_i^L \leq g_i^L(x)$.

By the rule of operation of interval numbers, objective function $f(x) = \left[f^L(x), f^U(x)\right]$ is an interval number. Let $f = \left[f^w, f^m\right]$, because of $x_i \geq 0 (i = 1, 2, ..., n)$, $f^w = \left(f^U(x) - f^L(x)\right)/2$ and $f^m = \left(f^L(x) + f^U(x)\right)/2$, take maximum value range inequality and minimum value range inequalities as constraint conditions in respect to objective function $f^L(x)$ *and* $f^U(x)$, then the reduced form of the interval nonlinear programming problem (5) is the following multi-objective nonlinear programming problem :

$Max\ f^{w} = \left(f^{U}(x) - f^{L}(x)\right)/2$

$Max\ f^{m} = \left(f^{L}(x) + f^{U}(x)\right)/2$

Subject to

$g_i^L(x) \ge G_i^L$

$g_i^U(x) \le G_i^U$

$x \ge 0$

The above multi-objective nonlinear programming problem can be solved by any multi-objective programming technique such as fuzzy technique, weighted sum method etc.

4. Reliability assessment through extension principle

Let $R_s(r_1, r_2, ..., r_n)$ be the reliability functions of a system having n units with components reliability $r_1, r_2, ..., r_n$; may connected in series, parallel, series-parallel, bridge, or any other configurations. Let components reliability be fuzzy numbers $\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n$ with their corresponding $\alpha-$level sets denoted by $\left[r_1^L(\alpha), r_1^U(\alpha)\right], \left[r_2^L(\alpha), r_2^U(\alpha)\right], ..., \left[r_n^L(\alpha), r_n^U(\alpha)\right]$. Then the $\alpha-$level set of the system reliability is represented by $\left[R_s(r_1, r_2, ...r_n)\right]_\alpha = R_s\left(r_1(\alpha), r_2(\alpha), ..., r_n(\alpha)\right)$ where $r_1(\alpha), r_2(\alpha), ..., r_n(\alpha)$ is $\alpha-$level set of the fuzzy reliability of $r_1, r_2, ..., r_n$ respectively. i.e. $r_1(\alpha) = \left[r_1^L(\alpha), r_1^U(\alpha)\right], r_2(\alpha) = \left[r_2^L(\alpha), r_2^U(\alpha)\right], ..., r_n(\alpha) = \left[r_n^L(\alpha), r_n^U(\alpha)\right]$.

4.1 Reliability assessment of bridge system using extension principle

Let us consider the bridge system whose components reliability are fuzzy in nature. The system reliability of bridge system (3) can be evaluated in interval by extension principle. The procedure to evaluate of fuzzy system reliability of bridge system using Zadeh's extension principle as follows; we have $R_s : R \times R \times R \times R \times R \to R$ where $\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4, \tilde{r}_5$ are n fuzzy numbers in R_1, R_2, R_3, R_4, R_5 and $\tilde{R}_s \subseteq R$ is given by:

$\tilde{R}_s = \left\{\left(t, \mu_{\tilde{R}_s}(t)\right)\right\}$, where $\mu_{\tilde{R}_s}(t) = Sup\left\{\min\left(\mu_{\tilde{r}_1}(t), \mu_{\tilde{r}_2}(t), ..., \mu_{\tilde{r}_5}(t)\right) : t = R_s\left(r_1, r_2, r_3, r_4, r_5\right)\right\}$

here $\alpha-cut$ of the system reliability $\tilde{R}_s$ is $R_s(\alpha) = \left[R_s^L(\alpha), R_s^U(\alpha)\right]$, such that $R_s^L(\alpha) = \min\left\{R_s(r_1, r_2, r_3, r_4, r_5) : r_i \in \left[r_i^L(\alpha), r_i^U(\alpha)\right] \forall i = 1,2,3,4,5\right\}$ and
$R_s^U(\alpha) = \max\left\{R_s(r_1, r_2, r_3, r_4, r_5) : r_i \in \left[r_i^L(\alpha), r_i^U(\alpha)\right] \forall i = 1,2,3,4,5\right\}$.

Consider the reliability of the components is TFN such as $\tilde{r}_i = (r_{i1}, r_{i2}, r_{i3})\ for\ i = 1,2,3,4,5$. Now taking $\alpha - cut$ of the TFN as follows $r_i(\alpha) = [r_{i1} + \alpha(r_{i2} - r_{i1}), r_{i3} - \alpha(r_{i3} - r_{i2})] = [r_{i1}(\alpha), r_{i2}(\alpha)]$ for i=1,2,3,4,5.

Again $R_s(r_1, r_2, r_3, r_4, r_5) = 2r_1r_2r_3r_4r_5 - r_1r_3r_4r_5 - r_2r_3r_4r_5 - r_1r_2r_3r_5 - r_1r_2r_4r_5 - r_1r_2r_3r_4 + r_1r_4r_5 + r_2r_3r_5 + r_1r_3 + r_2r_4$ as $\frac{dR_s}{dr_i} > 0 \forall i = 1,2,3,4,5\ as\ 0 < r_i < 1$, hence $\tilde{R}_s(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4, \tilde{r}_5)$ is an increasing function of $0 < r_i < 1\ \forall i = 1,2,3,4,5$, so the $\alpha - cut$ of the system reliability is $R_s(\alpha) = [R_s^L(\alpha), R_s^U(\alpha)]$ where:

$$\begin{aligned} R_s^L(\alpha) = {} & 2r_{11}(\alpha)r_{21}(\alpha)r_{31}(\alpha)r_{41}(\alpha)r_{51}(\alpha) - r_{11}(\alpha)r_{31}(\alpha)r_{41}(\alpha)r_{51}(\alpha) - r_{21}(\alpha)r_{31}(\alpha)r_{41}(\alpha)r_{51}(\alpha) \\ & - r_{11}(\alpha)r_{21}(\alpha)r_{31}(\alpha)r_{51}(\alpha) - r_{11}(\alpha)r_{21}(\alpha)r_{41}(\alpha)r_{51}(\alpha) - r_{11}(\alpha)r_{21}(\alpha)r_{31}(\alpha)r_{41}(\alpha) \\ & + r_{11}(\alpha)r_{41}(\alpha)r_{51}(\alpha) + r_{21}(\alpha)r_{31}(\alpha)r_{51}(\alpha) + r_{11}(\alpha)r_{31}(\alpha) + r_{21}(\alpha)r_{41}(\alpha) \end{aligned}$$

and

$$\begin{aligned} R_s^U(\alpha) = {} & 2r_{12}(\alpha)r_{22}(\alpha)r_{32}(\alpha)r_{42}(\alpha)r_{52}(\alpha) - r_{12}(\alpha)r_{32}(\alpha)r_{42}(\alpha)r_{52}(\alpha) - r_{22}(\alpha)r_{32}(\alpha)r_{42}(\alpha)r_{52}(\alpha) \\ & - r_{12}(\alpha)r_{22}(\alpha)r_{32}(\alpha)r_{52}(\alpha) - r_{12}(\alpha)r_{22}(\alpha)r_{42}(\alpha)r_{52}(\alpha) - r_{12}(\alpha)r_{22}(\alpha)r_{32}(\alpha)r_{42}(\alpha) \\ & + r_{12}(\alpha)r_{42}(\alpha)r_{52}(\alpha) + r_{22}(\alpha)r_{32}(\alpha)r_{52}(\alpha) + r_{12}(\alpha)r_{32}(\alpha) + r_{22}(\alpha)r_{42}(\alpha) \end{aligned}$$

The above assessment of system reliability becomes a nonlinear fuzzy number $\tilde{R}_s$ with its $\alpha - cut$ $R_s(\alpha)$ is $[R_s^L(\alpha), R_s^U(\alpha)]$.

4.2 Approximation of bridge system reliability and corresponding divergences

Since the components reliability is considered as TFN so the system reliability can be approximated to TFN [15]. Accordingly system reliability is derived as approximated TFN from the above assessment of the system reliability. Hence it has left divergence $\varepsilon_L(\alpha)$ and right divergence $\varepsilon_R(\alpha)$. So it is desired to find out the maximum divergence for the left $\varepsilon_L^*(\alpha^*)$ and right $\varepsilon_R^*(\alpha^*)$ spread of the TFN shaped system reliability. The approximate system reliability is in the form of TFN with membership is as follows:

$$\mu_{\tilde{R}_s}(x) = \begin{cases} R_s^{L-1}(x) & for\ a \le x \le b \\ R_s^{U-1}(x) & for\ b \le x \le c \\ 0 & otherwise \end{cases}$$

Where $a = \inf_{\forall\alpha\in[0,1]} R_s^L(\alpha)$, $b = \sup_{\forall\alpha\in[0,1]} R_s^L(\alpha) = \inf_{\forall\alpha\in[0,1]} R_s^U(\alpha)$, and $c = \sup_{\forall\alpha\in[0,1]} R_s^U(\alpha)$.

To find the left divergence for approximation of system reliability, the equation of left spread of the approximated TFN is $R_{SA}^{L}(\alpha) = a + (b-a)\alpha$ and the corresponding left divergence is $\varepsilon_L(\alpha) = R_s^L(\alpha) - R_{SA}^L(\alpha)$. From this it is easy to find out the maximum left divergence for the left spread of the TFN shaped system reliability of the bridge system.
Similarly, for the right divergence is $\varepsilon_R(\alpha) = R_s^U(\alpha) - R_{SA}^U(\alpha)$ where the equation of right spread of the approximated TFN is $R_{SA}^R(\alpha) = c - (c-b)\alpha$. Here also we can find the maximum right divergence.

5. Numerical exposure

For numerical illustration consider the components reliability of the five components bridge system as triangular fuzzy number given as follows $\tilde{r}_1 = (0.6, 0.8, 0.95), \tilde{r}_2 = (0.65, 0.85, .0.95)$, $\tilde{r}_3 = (0.75, 0.8, 0.9), \tilde{r}_4 = (0.7, 0.85, 0.95)$, and $\tilde{r}_5 = (0.85, 0.9, 0.95)$
The $\alpha - cut$ of the above triangular fuzzy number are $r_1(\alpha) = [0.6 + 0.2\alpha, 0.95 - 0.15\alpha]$, $r_2(\alpha) = [0.65 + 0.2\alpha, 0.9 - 0.1\alpha]$, $r_3(\alpha) = [0.75 + 0.05\alpha, 0.9 - 0.1\alpha]$, $r_4(\alpha) = [0.7 + 0.15\alpha, 0.95 - 0.1\alpha]$ and $r_5(\alpha) = [0.85 + 0.05\alpha, 0.95 - 0.05\alpha]$

5.1 Reliability of bridge system by extension principal and approximation method

Using the procedure of section 4.1 we get the following expression for the bridge system $R_s^L(\alpha) = 0.00003\alpha^5 + 0.0004375\alpha^4 - 0.00464625\alpha^3 - 0.0356\alpha^2 + 0.19538625\alpha + 0.7812125$ and $R_s^U(\alpha) = 0.9894725 - 0.0339375\alpha - 0.01877625\alpha^2 - 0.000145\alpha^3 + 0.00021375\alpha^4 - 0.0000075\alpha^5$.
The membership function of the system reliability of the bridge system can be reduced to an approximated TFN, whose membership function is:

$$\mu_{\tilde{R}_s}(x) = \begin{cases} (x - 0.7812125)/0.1556075 & for\ 0.7812125 \le x \le 0.93682 \\ (0.9894725 - x)/0.0526525 & for\ 0.93682 \le x \le 0.9894725 \\ 0 & otherwise \end{cases}$$

The figure-3 shows the system reliability graph by Zadeh's extension principal and the corresponding system reliability by approximated TFN graph.
For above approximated TFN, now finding the maximum divergence for the left and right spread of the TFN of the system reliability

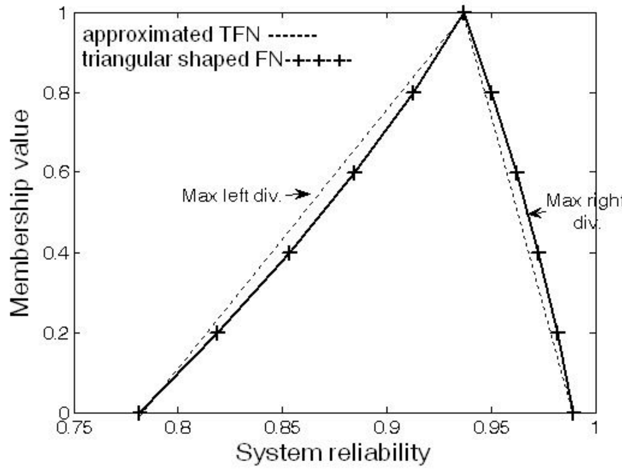

Figure 3. Membership function of reliability for bridge system.

5.1.1 Left divergence assessment for approximation of system reliability

The left spread of the TFN is $R_{SA}^{L}(\alpha)=0.1556075\alpha+0.7812125$ and let the left divergence is:

$\varepsilon_L(\alpha)$ which is given by

$$\varepsilon_L(\alpha)=R_s^L(\alpha)-R_{SA}^L(\alpha)$$
$$=0.00003\alpha^5+0.0004375\alpha^4-0.00464625\alpha^3-0.0356\alpha^2+0.03977875\alpha$$

To find out the maximum of $\varepsilon_L(\alpha)$, $\frac{d}{d\alpha}\varepsilon_L(\alpha)=0$ gives $\alpha^*=0.5109951$ at which maximum left divergence is $\varepsilon_L^*(\alpha^*)=0.01044195$

5.1.2 Right divergence assessment for approximation of system reliability

The right spread of the TFN is $R_{SA}^{R}(\alpha)=0.9894725-0.0526525\alpha$ and let the right divergence is $\varepsilon_R(\alpha)$ which is given by:

$$\varepsilon_R(\alpha)=R_s^R(\alpha)-R_{SA}^R(\alpha)$$
$$=-0.0000075\alpha^5+0.00021375\alpha^4-0.000145\alpha^3-0.01877625\alpha^2+0.018715\alpha$$

For find out the maximum of $\varepsilon_R(\alpha)$, $\frac{d}{d\alpha}\varepsilon_R(\alpha)=0$ gives $\alpha^*=0.4982479$ at which maximum right divergence is $\varepsilon_R^*(\alpha^*)=0.004658495$

5.2 Reliability evaluation by interval nonlinear programming of bridge system

The reliability optimization model reduces to an interval nonlinear programming problem with objective for maximize of system reliability and cost constraint after taking $\alpha-cut$ as follows:

Maximize $R_s(\alpha)=\left[R_s^L(\alpha),R_s^U(\alpha)\right]$ (6)

Subject to $\left[C_s^L(\alpha), C_s^U(\alpha)\right] \subseteq \left[C^L, C^U\right]$

$0 \le \alpha \le 1$

Where $R_s^L(\alpha) = 0.00003\alpha^5 + 0.0004375\alpha^4 - 0.00464625\alpha^3 - 0.0356\alpha^2 + 0.19538625\alpha + 0.7812125$,
$R_s^U(\alpha) = 0.9894725 - 0.0339375\alpha - 0.01877625\alpha^2 - 0.000145\alpha^3 + 0.00021375\alpha^4 - 0.0000075\alpha^5$,
$C_s^L(\alpha) = \sum_{i=1}^{5} a_i \exp\left(\frac{b_i}{1 - r_i^L(\alpha)}\right)$, $C_s^U(\alpha) = \sum_{i=1}^{5} a_i \exp\left(\frac{b_i}{1 - r_i^U(\alpha)}\right)$, $C^L = 50050$, $C^U = 50175$ and shape parameter are $a = 10000,\ and\ b = 0.0003$

The solution of the above problem is the Pareto optimal solution of the following multi-objective optimization problem:

$Max\, R_s^m(\alpha)$

$Max\, R_s^w(\alpha)$

Subject to

$C_s^L(\alpha) \ge C^L$

$C_s^U(\alpha) \le C^U$

$0 \le \alpha \le 1$

Where $R_s^m(\alpha) = \left(R_s^L(\alpha) + R_s^U(\alpha)\right)/2$ and $R_s^w(\alpha) = \left(R_s^U(\alpha) - R_s^L(\alpha)\right)/2$ are maximum and minimum value range inequalities with respect to objective functions $R_s^L(\alpha)\ and\ R_s^U(\alpha)$.

Solution of the above problem by fuzzy multi-objective optimization technique is $\alpha^* = 0.3069926$, and the system reliability is $R_s^*(\alpha^*) = [0.9802179,\ 0.8261024]$.

5.3 Sensitivity analysis

We have presented sensitivity analysis of reliability of bridge system by considering different interval value of the cost constraint goal. It is clear from the table-1 that reliability is proportional to the upper limit of the cost interval up to certain level which is expected. Reliability remain same for decreasing of lower bound of the cost interval, but for increasing of lower limit of the cost interval, lower and upper bound of reliability is increasing and decreasing respectively.

Table 1. Sensitivity analysis for different interval of cost constraint goal

α^*	$R_s^*(\alpha^*) = \left[R_s^U(\alpha^*), R_s^L(\alpha^*)\right]$	C^L	C^U
0.4493225	[0.9823291, 0.8170934]	50050	50185
0.3069926	**[0.9802179, 0.8261024]**	**50050**	**50175**
0.1457913	[0.9776871, 0.8361662]	50050	50165
0.3069926	[0.9802179, 0.8261024]	50030	50175
0.3069926	[0.9802179, 0.8261024]	50040	50175
0.3069926	**[0.9802179, 0.8261024]**	**50050**	**50175**
0.2317199	[0.9790549, 0.8308205]	50065	50175
0.0472626	[0.9760648, 0.8422417]	50067	50175

6. Conclusion

In uncertain conditions the reliability of components are taken as fuzzy number. Hence the system reliability is also a fuzzy number and belongs to an interval. Under the interval cost constrained goal, the problem becomes interval nonlinear programming and it is transformed into an equivalent multi-objective problem to compute the system reliability.

Here we have considered the bridge system in fuzzy environment to present the proposed method. The reliability of components is taken as triangular fuzzy number. We have considered bridge network for instance of complex system as it has widely used in a system design. Also we have introduced interval nonlinear programming and its solution procedure. The interval nonlinear programming problem transformed interval non linear programming problem into an equivalent multi-objective problem and then solve it by multi-objective optimization technique. The interval nonlinear programming is used when interval objective function and constraint functions and constraint goal is in interval. The procedure have presented in this paper can be used in many practical engineering models on reliability or any other filed which associated with uncertainty.

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