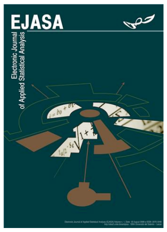

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STOCHASTIC MODELING OF THE GRADING PATTERN IN PRESENCE OF THE ENVIRONMENTAL PARAMETER

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Abstract: *This paper deals with the stochastic modeling of grading pattern, where grades are supposed to be influenced by environmental parameter 'c' (say),* ($0 \leq c \leq 1$). *The model discussed here is an extension of Wang's model (1981). The correlation coefficient between the two consecutive states is showed to be a function of the environmental parameter 'c'. The operating characteristics namely, first passage time distributions, mean waiting time in a particular grade are obtained. Also numerical examples are worked out to study the limiting behavior of the change.*

Keywords: *Grade, transition probability matrix, mean recurrence time, Markov chain, Eigen vector.*

1. Introduction

Neither two students' nor two schools are identical. Students' differ in gender, culture, religion, language, home environment, financial status of parents e.t.c., whereas the schools differ in size of students', quality of teacher, infrastructure, location of the school, aid provided by the government e.t.c.. Obviously performance of the students' measured in terms of scores or grades obtained by them in examinations, which varies from student to student and school to school. The variability in scores is a function of social climate which has to be studied and analyzed scientifically. The history of analyzing the students' performance is as old as history of education. However formal presentation of analysis started around early thirties of the 20th century.
A test or an examination is an assessment often administered on paper or on the computer, intended to measure the test-takers' or respondents' (often a student) knowledge, skills, aptitudes

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or classification in many other topics. Tests are often used in education, professional certification, counseling and many other fields. The measurement, that is the goal of testing is called a test score, and is "a summary of the evidence contained in an examinee's responses to the items of a test that are related to the construct or constructs being measured." Test scores are interpreted with regards to a norm or criterion or occasionally both.

The study of grading pattern in presence of environmental parameters is not new. Hu & Hossler (2000) eexamined and evaluated factors affecting students' behavior and performance in college or university, such as South Carolina State University [7]. The Chi-square tests and Likelihood ratio test statistics reveal that satisfaction with academic environment and services as well as the precedent high school achievements are significantly correlated with college performance. Educational standards are continuously revised and often raised while competition for high-achieving students' has intensified. A stochastic modeling on grading pattern has been analyzed by Sarma and Sarmah (1999) to study various performance measures [10]. 'Evaluation of factors related to students' performance in a distance-learning business communication course' studied by Cheung, *et al.* (2002) [4]. Karemera *et al.* (2003) analyzed on the effects of academic environment and background characteristics on student satisfaction and performance [5]. Biktimirnov *et al.* (2008) authored an article on 'relation between use of online support materials and student' [3]. Hadsell and Lester (2009) analyze the effect of quiz timing on examination performance [6].

This paper deals with stochastic modeling of grading pattern where grades are assumed to be influenced by environment parameter 'c' (say)($0 \leq c \leq 1$). Wang (1981) presented an article [12] on Markov-Bernoulli chain, where 'c' was found to be nothing but correlation coefficient between two consecutive states of a two state Markov chain. Here in this paper Wang's model has been extended to study the dependency of grading pattern between Matriculation or school leaving (i.e., X^{th} standard) and Higher Secondary examinations (i.e., XII^{th} standard). Section 2 presents the description of real life dataset; section 3 of this paper shows the model and methodology along with the assumptions. Section 4 is about operating characteristics of the grading pattern whereas section 5 deals with correlation coefficient between two consecutive states, when students' are subject to several tests at regular interval of time. Behavior of operating characteristics is studied numerically from a set of real life data.

2. Research Design and Data Set

A 'Pilot-Survey' has been conducted in and around Tripura (North-East India: our study area) taking 25 (twenty five) schools to analyze the examination results of Matriculation and Higher Secondary [consider science stream] students. The marks of total 538 students [male (293) and female (245)] those appear in both the examination from the same schools.

3. Model and Methodology

A grade is defined as the Teachers' standardized evaluation of students' performance. In some cases, evaluation can be expressed quantifiable way, and calculated into a numeric Grade Point

Average (GPA). To understand the probabilistic pattern of grades, the following assumptions, reported in the next subparagraphs, are made.

3.1 Assumptions

a) A student is evaluated at regular interval of time and each time point the grade of the student is recorded.

b) The present grade of the student is dependent on immediate previous grade i.e. future course is decided by the immediate past position, for example see [10] and [11].

c) The range [0, 100] of scores "S" is partitioned into three non-overlapping sets viz. S_0, S_1, and S_2 such that, $S_0 = [M_0, 100]$, $S_1 = [M_1, M_0)$ and $S_2 = (M_1, 0]$ where $0 \leq M_1 < M_0 \leq 100$. M_0 & M_1 are the lower limits of the top and middle grade respectively.

d) The grade of a student is assumed to be *i* if $S \in S_i$ for $i = 0,1,2$

Now let $X_n = i$ for $i = 0,1,2$ be the grade of a student at the n^{th} test, where $n = 1,2,3,....$ The consequence of assumptions **a** to **d** of section 3.1. states that $\{X_n,\ n = 1,2,...\}$ follows a Markov chain with state space $S = \{0,1,2\}$, transition probability matrix $\mathbf{P} = (P_{ij})$, initial distribution (P_0, P_1, P_2), such that $P_0 + P_1 + P_2 = 1$ and $P_r(X_0 = i) = P_i$ with $0 < P_i$, $P_{ij} < 1$ for $i, j = 0,1,2$. Where X_0 = initial states of the chain. The transition probability matrix of the chain may be written as:

$$\mathbf{P} = \begin{bmatrix} 1-a(1-c) & a(1-c) & 0 \\ \frac{b}{2}(1-c) & 1-b(1-c) & \frac{b}{2}(1-c) \\ 0 & k(1-c) & 1-k(1-c) \end{bmatrix} \qquad (1)$$

where *a*, *b*, *k* are all real numbers lies between 0 & 1, and *c* is the environmental parameter, with the same range, i.e., ($0 \leq c \leq 1$).

For *c*=0 *i.e.*, students are free from the effect of environmental parameter, then the above matrix (1) reduces to:

$$\mathbf{P} = \begin{bmatrix} 1-a & a & 0 \\ \frac{b}{2} & 1-b & \frac{b}{2} \\ 0 & k & 1-k \end{bmatrix}$$

which is the matrix representing Markovian movement of grades for matriculation and higher secondary examinations developed in [10] using real life data.

For c=1, i.e. students are under maximum influence of environmental parameter, then the t.p.m. turns out to be a diagonal matrix viz. (1,1,1) and in such situation the matrix (1) may be written as: $\mathbf{P} = diag(1,1,1) + (1-c)\cdot\mathbf{P}^*$

Where:

$$\mathbf{P}^* = \begin{bmatrix} a & -a & 0 \\ -b/2 & b & -b/2 \\ 0 & -k & k \end{bmatrix}$$

Though a students' performance is highly influenced by the environmental parameter, with constant afford, it is expected that this affect may be reduced to a great extent with an increase in number of tests, which are to be faced by the students' during the period of course.

By using the theory of characteristic roots it is possible to obtain $P^{(n)}$ for higher order transition probabilities such that,

$$P^{(n)} = \sum_{j=0}^{2} \alpha_j^n c_j X_j Y_j^{/}$$

Where $(\alpha) = diag(\alpha_0\ \alpha_1\ \alpha_2) = diag(1\,1\,1) - (1-c)\cdot(\lambda)$

With $\lambda = diag(\lambda_0 \lambda_2 \lambda_2)$ being the characteristic root of **P***.

and $C_j = \dfrac{1}{(Y_j^{/} X_j)}$ for $j = 0,1,2$.

Specifically,

$$\alpha_0 = 1-(1-c)\lambda_0 = 1-0 = 1$$

$$\alpha_1 = 1-(1-c)\lambda_1 = 1-(1-c)\frac{(a+b+k)+\sqrt{(a-k)^2+b^2}}{2}$$

$$\alpha_2 = 1-(1-c)\lambda_2 = 1-(1-c)\frac{(a+b+k)-\sqrt{(a-k)^2+b^2}}{2}$$

It may be shown that one element of the diagonal matrix is one, whereas other two elements are less than one (Page No. 101) from [8].

Hence, $\alpha_1^n \to 0$, $\alpha_2^n \to 0$ as $n \to \propto$ whereas $|\alpha_1| < 1$ and $|\alpha_2| < 1$.

Therefore, $\lim_{n\to\infty} P^{(n)} = \lim_{n\to\infty} \sum_{j='}^{2} \alpha_j^n c_j \mathbf{X}_j \mathbf{Y}_j^{'} = \dfrac{bk}{bk+2ak+ab}(\beta)$

Where (β) is a 3×3 matrix, $\beta_{00} = 1 = \beta_{10} = \beta_{20}$, $\beta_{01} = 2a/b = \beta_{11} = \beta_{21}$ and $\beta_{02} = a/k = \beta_{12} = \beta_{22}$ which is independent of environmental parameter and is identical to the result obtained in [10].

4. Operating characteristics of the grading pattern

The chain defined above is an irreducible and aperiodic. The states of the chain are either transient or persistent which may be easily seen from the following arguments:

We have,

$$P_{ij}^{(n)} = \Pr\left(X_n = j / X_0 = i\right) = \Pr\left(\text{that the student reaches the state } j \text{ from state } i \text{ in } n \text{ transitions}\right)$$
$$F_{ij} = \Pr\left(\text{that a student starting with state } i \text{ ever reach the state } j\right)$$
$$f_{ij}^{(n)} = \Pr\left(\text{that a student starting with state } i \text{ visit the state } j \text{ for the first time in } n \text{ transitions}\right)$$

So clearly, $F_{ij} = \sum_{n=1}^{\infty} f_{ij}^{(n)}$ (2)

If i=j, then we have F_{ii}. Now:
i. If $F_{ii} = 1$, it is said to be persistent.
ii. If $F_{ii} < 1$, it is said to be transient.

4.1 Calculation of first passage time distribution for different grades

By applying the t.p.m. (1) in section 3, the following results obtained from equation no. (2), after putting i=j=0:

$$f_{00}^{(1)} = \left\{1 - a(1-c)\right\}$$
$$f_{00}^{(2)} = \frac{ab}{2}(1-c)^2$$
$$f_{00}^{(3)} = \frac{ab}{2}(1-c)^2\left\{1 - b(1-c)\right\}$$

Therefore, $F_{00} = \sum_{n=1}^{\infty} f_{ij}^{(n)} = 1 - \frac{a(1-c)}{2} < 1$ (after simplification). Hence the state '0'is transient. Again, for i=j=1, by equation no. (2):

$$f_{11}^{(1)} = \{1 - b(1-c)\}$$
$$f_{11}^{(2)} = \frac{b}{2}(1-c)^2(a+k)$$
$$f_{11}^{(3)} = \frac{b}{2}(1-c)^2\left\{a\left[1-a\left(1-c\right)\right]+k\left[1-k\left(1-c\right)\right]\right\}$$

So, $F_{11} = \sum_{n=1}^{\infty} f_{ij}^{(n)} = 1$ (after simplification)

This shows that the state '1' is persistent or recurrent. Similarly,

$$f_{22}^{(1)} = \{1 - k(1-c)\}$$
$$f_{22}^{(2)} = \frac{bk}{2}(1-c)^2$$
$$f_{22}^{(3)} = \frac{bk}{2}(1-c)^2\left\{1-b\left(1-c\right)\right\}$$

So, $F_{22} = \sum_{n=1}^{\infty} f_{ij}^{(n)} = 1 - \frac{k(1.c)}{2} < 1$ (after simplification)

i.e., the state '2' is also transient.

Also, it can be say that, there is no relation between the parameters *a*, *b*, *k*. Since students' performance measures vary from student to student, which is obvious from the transition probability matrix.

Let us check the above explanation by using the following transition probability matrix by (Obtained by Method of Maximum Likelihood Estimation) [1] and [2] from our real life data as,

$$\mathbf{P} = (P_{ij}) = \begin{bmatrix} 0.329 & 0.671 & 0 \\ 0.114 & 0.765 & 0.121 \\ 0 & 0.284 & 0.716 \end{bmatrix} \text{ where } i, j = 0,1,2.$$

Therefore,

$f_{00}^{(1)} = 0.329$, $f_{00}^{(2)} = 0.076$ and $f_{00}^{(3)} = 0.0585$

So, $F_{00} = 0.4635 < 1$. Hence the state '0' is transient. Also,

$f_{11}^{(1)} = 0.765$, $f_{11}^{(2)} = 0.1109$ and $f_{11}^{(3)} = 0.0498$

So, $F_{11} = 0.8957 \cong 1$

This shows that the state '1' is persistent or recurrent. Again,

$f_{22}^{(1)} = 0.716$, $f_{22}^{(2)} = 0.034$ and $f_{22}^{(3)} = 0.0263$

So, $F_{22} = 0.7763 < 1$
i.e., the state '2' is also transient.

4.2 Calculation of mean recurrence times of the chain for different grades

The mean first passage time for the above chain gives the time required for a student to visit a particular state provided he/she was in the same state initially.
The mean time required for a student starting with the grade i at the beginning of the course to come out with grade j is:

$$\mu_{ij} = \sum_{n=1}^{\infty} n f_{ij}^{(n)} \text{ ; } i, j = 0,1,2.$$

Where $f_{ij}^{(n)}$ is the probability that a student visits the state *j* for the first time in '*n*' tests provided, he / she enters the state *i* initially. Hence, the mean time required for a student starting with initial grade '0' to reach the same grade '0' is:

$$\mu_{00} = \left\{1 - a(1-c)\right\} + \frac{a}{2b}\left\{1 + b(1-c)\right\} \text{ (after simplification)}$$

Again the mean time required for a student starting with grade '1' to go back to grade '1' after n tests is given by,

$$\mu_{11} = 1 + \frac{b}{2ak}(a+k) < \infty \text{ (after simplification)}$$

This also shows that state '1' is non-null. Similarly, the mean time required for a student starting or entering the school initially with grade '2' to go back again to the same grade '2' at the end is,

$$\mu_{22}\left\{1 - k(1-c)\right\} + \frac{k}{2b}\left\{1 + b(1-c)\right\} \text{(after simplification)}$$

4.3 Expected number of times students' visits to different states

Let M_{ij}= Expected number of times a student visits the state *j* in a fixed number tests *n*, provided the student was in the state *i* initially. That is, if the student visits the state *j* for the first time in '*k*' tests, then,

$$M_{ij} = \frac{\sum_{k=1}^{n} f_{ij}^{(k)}}{1 - \sum_{k=1}^{n} f_{ij}^{(k)}}\ ;\ i, j = 0,1,2.$$

In this context following three cases were discussed.

<u>*Case I*</u>: If $i=j=0$, then

$$M_{00} = \frac{\left\{1 - a(1-c)\right\} + \frac{a(1-c)}{2}\left\{1 - \left[1 - b(1-c)\right]^{n-1}\right\}}{a(1-c) - \frac{a(1-c)}{2}\left\{1 - \left[1 - b(1-c)\right]^{n-1}\right\}} \text{ (after simplification)}$$

So, the expected number of times a student visits the grade '0' for a fixed number of tests n, can be easily obtained, provided the student starts with grade '0' initially.

<u>*Case II*</u>: If $i=j=1$, then

$$M_{11} = \frac{\left\{1 - b(1-c)\right\} + \frac{b(1-c)}{2}\left\{2 - \left[1 - a(1-c)\right]^{n-1} - \left[1 - k(1-c)\right]^{n-1}\right\}}{1 - \left\{\left\{1 - b(1-c)\right\} + \frac{b(1-c)}{2}\left\{2 - \left[1 - a(1-c)\right]^{n-1} - \left[1 - k(1-c)\right]^{n-1}\right\}\right\}} \text{ (after simplification)}$$

hence for $i= j= 1$, the expected number of times a student visits the grade '1', in a fixed number of tests can be obtained, if the student starts with grade '1' initially.

<u>*Case III*</u>: If $i=j=2$, then

$$M_{22} = \frac{\left\{1 - k(1-c)\right\} + \frac{k(1-c)}{2}\left\{1 - \left[1 - b(1-c)\right]^{n-1}\right\}}{k(1-c) - \frac{k(1-c)}{2}\left\{1 - \left[1 - b(1-c)\right]^{n-1}\right\}} \text{ (after simplification)}$$

Thus M_{22} gives us the expected number of times; the student visits the grade '2' in a fixed number of tests, if the student starts with grade '2' initially.
Then the transition probability matrix obtained from real life data is given by the method of maximum likelihood estimates due to [1] as:

$$\mathbf{P} = (P_{ij}) = \begin{bmatrix} 0.329 & 0.671 & 0 \\ 0.114 & 0.765 & 0.121 \\ 0 & 0.284 & 0.716 \end{bmatrix} \text{ where } i, j=0,1,2.$$

The range of '*c*' is determines such that, $Max(P_{ij}) < 1$ and hence this gives us $0 \le c \le 0.240$.Hence, by using the possible choices of *a*, *b*, *c* and *k* with different values of *n*, we can compute the values of M_{00}, M_{11} & M_{22} in the following Tables 1, Table 2 and Table 3 as:

Table 1. Numerical Values of M_{00}.

				n									
				1	2	3	4	5	6	7	8	9	10
SL. NO.	*a*	*b*	*c*	M_{00}									
1	0.671	0.228	0	0.4903	0.6821	0.8676	1.0414	1.1994	1.3392	1.4599	1.5619	1.6467	1.7161
2	0.685	0.233	0.02	0.4896	0.6816	0.8674	1.0413	1.1994	1.3393	1.4599	1.5619	1.6466	1.7158
3	0.706	0.240	0.05	0.4909	0.6828	0.8684	1.0423	1.2004	1.3402	1.4609	1.5631	1.6479	1.7173
4	0.729	0.248	0.08	0.4910	0.6830	0.8688	1.0428	1.2009	1.3409	1.4616	1.5637	1.6485	1.7178
5	0.746	0.253	0.10	0.4894	0.6808	0.8659	1.0394	1.1972	1.3368	1.4574	1.5594	1.6442	1.7136

Table 2. Numerical Values of M_{11}.

					n									
					1	2	3	4	5	6	7	8	9	10
SL. NO.	*a*	*b*	*c*	*k*	M_{11}									
1	0.025	0.228	0	0.284	3.3859	4.1874	4.9947	5.7793	6.5198	7.2036	7.8266	8.3912	8.9037	9.3725
2	0.035	0.233	0.02	0.290	3.3794	4.2089	5.0617	5.9112	6.7360	7.5221	8.2631	8.9587	9.6130	10.2327
3	0.055	0.240	0.05	0.299	3.3859	4.2725	5.2176	6.2002	7.2014	8.2067	9.2072	10.1992	11.1834	12.1638
4	0.060	0.248	0.08	0.309	3.3829	4.2789	5.2394	6.2444	7.2757	8.3189	9.3652	10.4105	11.4553	12.5034
5	0.065	0.253	0.10	0.316	3.3917	4.3005	5.2806	6.3134	7.3814	8.4708	9.5725	10.6825	11.8010	12.9313

Table 3. Numerical Values of M_{22}.

				n									
				1	2	3	4	5	6	7	8	9	10
SL. NO.	*b*	*c*	*k*	M_{22}									
1	0.228	0	0.284	2.5212	2.9742	3.4125	3.8231	4.1965	4.5268	4.8119	5.0530	5.2533	5.4172
2	0.233	0.02	0.290	2.5186	2.9721	3.4108	3.8217	4.1952	4.5255	4.8105	5.0514	5.2514	5.4149
3	0.240	0.05	0.299	2.5205	2.9735	3.4117	3.8223	4.1956	4.5258	4.8109	5.0519	5.2522	5.4161
4	0.248	0.08	0.309	2.5177	2.9706	3.4088	3.8193	4.1925	4.5225	4.8075	5.0483	5.2483	5.4119
5	0.253	0.10	0.316	2.5162	2.9679	3.4050	3.8146	4.1871	4.5167	4.8014	5.0422	5.2423	5.4062

5. Correlation coefficient between the two consecutive states

The correlation coefficient between two consecutive states viz. X_n and X_{n-1} is given by:

$$r = \frac{\text{cov}(X_n, X_{n-1})}{\sqrt{\text{var}(X_n) \cdot \text{var}(X_{n-1})}} = \frac{E(X_n, X_{n-1}) - E(X_n) \cdot E(X_{n-1})}{\sqrt{\text{var}(X_n)} \cdot \sqrt{\text{var}(X_{n-1})}}$$

To calculate $\text{var}(X_n)$ and $\text{var}(X_{n-1})$, let us define the matrix $\mathbf{W}_n$ as:

$\mathbf{W}_n = \begin{bmatrix} \mathbf{P}_n & \mathbf{Q}_n & \mathbf{R}_n \end{bmatrix}$ such that, $\mathbf{P}_n + \mathbf{Q}_n + \mathbf{R}_n = 1$ for n=0,1,2,…

with $\mathbf{W}_0 = \begin{bmatrix} \mathbf{P}_0 & \mathbf{Q}_0 & \mathbf{R}_0 \end{bmatrix}$ representing the initial distribution of the chain and

$P_n = P_r(X_n = 0)$, $Q_n = P_r(X_n = 1)$, and $R_n = P_r(X_n = 2)$

Now:

$$P_n = P_r(X_n = 0) = P_r(X_n = 0, X_{n-1} = 0) + P_r(X_n = 0, X_{n-1} = 1) + P_r(X_n = 0, X_{n-1} = 2)$$

$$\therefore P_n = P_{00}P_{n-1} + P_{10}Q_{n-1} + P_{20}R_{n-1} \quad (3)$$

Similarly,

$$\therefore Q_n = P_{01}P_{n-1} + P_{11}Q_{n-1} + P_{21}R_{n-1} \quad (4)$$

$$\therefore R_n = P_{02}P_{n-1} + P_{12}Q_{n-1} + P_{22}R_{n-1} \quad (5)$$

The equations (3), (4) and (5) together can be represented as:

$$W_n = W_{n-1} \cdot p = W_{n-2} \cdot p^2 = ... = W_0 \cdot p^n$$

Where W_0 is defined above.

Also, $E(X_n) = (Q_n + 2R_n)$; $E(X_n^2) = (Q_n + 4R_n)$

and $v(X_n) = Q_n(1 - Q_n) + 4R_n(1 - R_n) - 4Q_nR_n$.

Similarly, $E(X_{n-1}) = (Q_{n-1} + 2R_{n-1})$; $E(X_{n-1}^2) = (Q_{n-1} + 4R_{n-1})$

And $v(X_{n-1}) = Q_{n-1}(1 - Q_{n-1}) + 4R_{n-1}(1 - R_{n-1}) - 4Q_{n-1}R_{n-1}$

To obtain the expression for $\text{cov}(X_n, X_{n-1})$, let us define:

$Z_n = (X_n = i, X_{n-1} = j)$; i, j=0,1,2 and n=1,2,3,…

where Z_n's are random variables taking values greater than zero only when two consecutive states are (1,1), (1,2), (2,1) and (2,2) assuming values 1, 2, 3 and 4.

Let us define: $p_{ij}^k = p_{ik} \cdot p_{kj}$ for i, j=0,1,2 and k=1,2.
Also $a_{ij} = p_{ij}^k + 2p_{ij}^k$

Hence,

$$E(X_n, X_{n-1}) =$$
$$= P_{n-2}\left(p_{01}^{(1)} + 2p_{01}^{(2)} + 2p_{02}^{(1)} + 4p_{02}^{(2)}\right) + Q_{n-2}\left(p_{11}^{(1)} + 2p_{11}^{(2)} + 2p_{12}^{(1)} + 4p_{12}^{(2)}\right) + R_{n-2}\left(p_{21}^{(1)} + 2p_{21}^{(2)} + 2p_{22}^{(1)} + 4p_{22}^{(2)}\right)$$
$$= (P_{n-2}, Q_{n-2}, R_{n-2}) \cdot \begin{pmatrix} a_{01} + 2a_{02} \\ a_{11} + 2a_{12} \\ a_{21} + 2a_{22} \end{pmatrix}$$

$= (W_{n-2}) \cdot a$; where $n \geq 2$.

Where,

$a = \left(a_0, a_1, a_2\right)$

With $a_0 = a_{01} + 2a_{02}$, $a_1 = a_{11} + 2a_{12}$, $a_2 = a_{21} + 2a_{22}$ and $W_{n-2} = (P_{n-2}, Q_{n-2}, R_{n-2})$

Therefore the correlation coefficient:

$$r(X_n, X_{n-1}) = \frac{E(X_n, X_{n-1}) - E(X_n) \cdot E(X_{n-1})}{\sqrt{\text{var}(X_n)} \cdot \sqrt{\text{var}(X_{n-1})}}$$

$$\therefore r(X_n, X_{n-1}) = \frac{\xi}{\phi} \qquad (6)$$

Where,

$$\xi = (W_{n-2}) \cdot a - (Q_n + 2R_n) \cdot (Q_{n-1} + 2R_{n-1})$$

And

$$\phi = \sqrt{Q_n(1 - Q_n) + 4R_n(1 - R_n) - 4Q_nR_n} \cdot \sqrt{Q_{n-1}(1 - Q_{n-1}) + 4R_{n-1}(1 - R_{n-1}) - 4Q_{n-1}R_{n-1}}$$

Hence, for different values of n=1,2,3,…, P_n, Q_n and R_n, the values of $r(X_n, X_{n-1})$ may be computed numerically from the above equation number (6).

6. Conclusion

From the above discussion it may be observed that, the environmental parameter affects the grading pattern of examination considerably. However, in case of calculations of correlation coefficient we need some well-defined computer programs, which is beyond the scope of this paper and hence work in this direction is in-progress.

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