# Longevity Bonds: an application to the Italian annuity market

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**Abstract**: *The paper focuses on the securitization of longevity risk through mortality-linked securities. Alternative mortality-linked securities have been proposed in literature (see Cairns-Blake-Dowd (2006)) and among these we considered the longevity bond as the most appropriate to hedge longevity risk. The paper aim at comparing two different approaches developed in the actuarial literature for the pricing of mortality linked securities: the one based on a distortion operator as the Wang transform and the other based on the arbitrage-free pricing framework used for financial derivatives. We underline drawbacks and advantages of each method, with reference to the Italian population. Each approach is applied to the case study adopting a Lee-Carter log-bilinear model to represent the evolution of mortality. Later on, we calculate the risk adjusted market price of a longevity bond with a constant fixed coupon.*

**Keywords: Stochastic mortality, longevity risk, longevity bonds, annuity market**

## 1. Introduction

Pension plans and life insurance companies are usually exposed to longevity risk, the risk that mortality rates of their reference population might differ from that expected, affecting their pricing and reserving calculations. The unanticipated mortality improvements have most significance at higher ages, leading annuity providers to experience losses in their life annuity business. The annuity providers are also heavily exposed to interest-rate risk because their investment portfolios are predominantly fixed income.

Several authors (see, for example, Milevsky and Promislow (2001), Dahl (2004), and Biffis (2005)) have observed that there are important similarities between the force of mortality and interest rates. In particular, both are positive processes, have term structures, and are fundamentally stochastic in nature. It is now widely accepted that the similarities between mortality and interest rate risks allows to model mortality risks and to price mortality-related instruments using adaptations of the arbitrage-free (or risk-neutral) pricing frameworks developed for interest-rate derivatives.

This paper deals with the securitization of longevity risk through the mortality-linked securities. Securitization offers great opportunities for hedging the longevity risk for annuity providers. It consists in isolating the cash flows linked to the longevity risk, repackaging them into cash flows traded on capital markets. Several mortality-linked securities have been proposed in literature (see Cairns-Blake-Dowd (2006)), among these longevity bonds can be considered more appropriate to hedge longevity risk. When considering a cohort of individuals born in the same year, the longevity risk can be hedged with longevity bonds having coupons proportional to the number of cohort' survivors at each year.

## 2. Stochastic modelling for mortality

When concerning the mortality models, a lot of models have been developed to forecast mortality rates over time. One of the most famous has been proposed by Lee and Carter (1992). We refer here to the extention of such a model proposed by Brouhns-Denuit-Vermont (2002).
The central death rates $m_x(t)$ at age x and time t. follows the model suggested by Lee and Carter:

$$\ln m_x(t) = \alpha_x + \beta_x k_t \tag{1}$$

where the parameters are subject to the following constraints: $\sum_t k_t = 0$ and $\sum_x \beta_x = 1$.

$\alpha_x$ refers to the average shape across ages of the log of mortality schedule; $\beta_x$ describes the pattern of deviations from the previous age profile, as the parameter $k_t$ changes, and $k_t$ can be seen as an index of the general level of mortality over time.
To forecast future death rates, Lee and Carter (1992) assume that $\alpha_x$ and $\beta_x$ remain constant over time and forecast future values of the time factor $k_t$, intrinsically viewed as a stochastic process, using a standard univariate time series model. Box-Jenkins identification procedures are used here to estimate and forecast the Autoregressive Integrated Moving Average model (ARIMA). With the mere extrapolation of the time factor $k_t$, it is possible to forecast the entire matrix of future death rates.
Considering the higher variability of the observed death rates, at ages with a smaller number of deaths, Brouhns-Denuit-Vermont assumed a Poisson distribution for the random component, against the assumed Normal distribution in the original Lee-Carter model. Therefore, they assumed that the number of deaths, $D_x(t)$, is a random variable following a Poisson distribution:

$$D_x(t) \sim Poisson(N_x(t)m_x(t)) = \frac{[N_x(t)m_x(t)]^{D_x(t)} e^{-[N_x(t)m_x(t)]}}{D_x(t)!} \tag{2}$$

where $N_x(t)$ is the mid-year population observed at age $x$ and time $t$.

## 3. Longevity bond pricing models

We focus on the longevity bonds pricing. Prices are calculated as the expected present value of future cash flows under a risk-adjusted probability measure. Since the market of mortality-linked securities is incomplete, the risk-adjusted measure cannot be estimated consistently with observed market prices of longevity bonds.
To price the longevity bond we consider two different pricing methods: the risk-neutral approach suggested by Milevsky and Promislow (2001), and the distortion approach suggested by Wang (2002).
The first one consists in adapting the arbitrage-free pricing framework of interest-rate derivatives to the valuation and securitization of mortality risk. The price of mortality-linked securities is therefore given by the expected present value of future cash flows under a risk-neutral probability measure $\mathbb{Q}$ (or equivalent martingale measure). The mortality dynamic is specified under a risk-adjusted pricing measure $\mathbb{Q}$ that is equivalent to, in the probabilistic sense, the current real-world measure $\mathbb{P}$ (often called physical measure). Example of this approach are the papers of Milevsky and Promislow (2001), Dahl (2004), Cairns-Blake-Dowd (2006), Biffis and Denuit (2006) and Biffis-Denuit-Devolder (2005). Here the main problem is how to find a risk-neutral pricing measure, given the lack of mortality-linked securities.

 

The other approach is based on a distortion operator - the Wang transform (see Wang (2002)) - that distorts the distribution of projected death probability to generate risk-adjusted expected values that can be discounted at the risk-free rate. Example of this approach are reported in the papers of Lin and Cox (2005), Cox, Lin andWang (2006) and Denuit, Devolder and Goderniaux (2007).

## 4. Numerical simulation and conclusions

The models above described are here applied to the Italian population. We consider two cases:

1. the longevity bond is built on the forecast of the Italian insured mortality, obtained from the general population data and then applying self-selection factors taken from ANIA (the National Association of Insurance Companies)
2. the longevity bond is built on the forecasts of the Italian general population mortality

Note that when using population data and not specific of the insurer annuitants (case 2) a basis risk is introduced. Thus, the hedge does not perfectly match the longevity risk of the annuity provider. Conversely, in case 1 the basis risk is strongly mitigated by the self-selection factors.

The different prices obtained with the risk-neutral approach and the distortion approach are the direct consequence of the difference in death probabilities. The small value of premium P in the case 1 is due to the point wise estimate of risk-neutral death probabilities that are greater than the physical ones.

Note that when longevity bond is built on general population (case 2), premium P incorporates both longevity risk and annuitants self-selection.

The choice of the reference population is a critical issue when constructing longevity bonds. To provide a real hedge the bond's cash flows have to match the payments to the annuitants from the annuity provider. Besides, the reference population must be chosen taking into account both hedging needs and speculative interest.

To this purpose we analyse two different cases: in the first one the longevity bond is built on the insured population realizing the hedging needs of the annuity provider, in the second case the bond is built on the general population and then more adequate for speculative purposes. On the other hand, in case 2 the bond requires a greater premium P and introduces basis risk. Our analysis shows that the lack of a secondary annuity market in Italy causes problems in the estimate of the market price of longevity risk giving contradictory pricing results. Therefore, it is difficult to establish the best pricing method. Distortion approach sounds more appropriate for incomplete markets, but it is not coherent with pricing model mark to market applied to other securities.

## Bibliography

ANIA (2005), IPS55 Basi demografiche per le assicurazioni di rendita, Documento di consultazione, Roma.

Biffis E. (2005), Affine processes for dynamic mortality and actuarial valuations, Insurance: Mathematics and Economics, 37, 443-468.

Biffis E., Denuit M., Devolder P. (2005), Stochastic Mortality Under Measure Changes, Discussion Paper PI-051, Publisher: Pensions Institute, London.

Biffis E., Denuit M. (2006), Lee-Carter goes risk-neutral: An application to the Italian annuity market, Giornale dell'Istituto Italiano degli Attuari, LXIX, 33-53.

Blake D., Cairns A. J. G., Dowd, K. (2006), Living with mortality: longevity bonds and other mortality-linked Securities, Presented to the Faculty of Actuaries, 16 January 2006.

Blake D., Cairns A., Dowd K., MacMinn R. (2006), Longevity Bonds: Financial Engineering, Valuation and Hedging, Pension Institute, London.

Brouhns N., and Denuit M., Vermunt J.K. (2002), A Poisson log-bilinear approach to the construction of projected life tables, Insurance: Mathematics and Economics, 31, 373-393.

Cairns A. J. G., Blake D., Dowd K. (2006): Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk, Astin Bulletin, 36, 79-120.

Cox S.H., Lin Y., Wang, S. (2006), Multivariate exponential tilting and pricing implications for mortality securitization, The Journal of Risk and Insurance, 73(4), 719-736.

Dahl M. (2004), Stochastic mortality in life insurance: market reserves and mortality-linked insurance contracts, Insurance: Mathematics and Economics, 35, 113-136.

Denuit M., Devolder P., Goderniaux A. C. (2007), Securitization of Longevity Risk: Pricing Survivor Bonds with Wang Transform in the Lee-Carter Framework, The Journal of Risk and Insurance, 74(1), 87-113.

Efron B. (1979), Bootstrap methods: another look for the jack-knife, The Annals of Statistics, 7 1-26.

Goodman L.A., Simple models for the analysis of association in cross-classifications having ordered categories, Journal of the American Statistical Association, 74, 537-552.

Koissi M.C., Shapiro A.F. (2006), Evaluating and extending the Lee-Carter model for mortality forecasting: Bootstrap confidence interval, Insurance: Mathematics and Economics, 38, 1-20.

Lee R.D., Carter L.R. (1992), Modelling and forecasting U.S. mortality, Journal of the American Statistical Association, 87, 659-675.

Levantesi S., Torri T. (2008), Setting the hedge of longevity risk through securitization, Proceedings of the 10th Italian-Spanish Congress of Financial and Actuarial Mathematics, Cagliari.

Lin Y., Cox S.H. (2005), Securitization of Mortality Risks in Life Annuities, The Journal of Risk and Insurance, 72, 227-252.

Milevsky M.A., Promislow S.D. (2001), Mortality derivatives and the option to annuitise, Insurance: Mathematics and Economics, 29, 299-318.

Wang S. (2002), A universal framework for pricing financial and insurance risks, ASTIN Bulletin, 32.

 